12-08-2010

Review for Final

Your final exam will be similar to exams I-III. The exam will have 10 problems worth 10 points each. It is a cumulative exam. Here is a list of topics:

- From chapter 1: Know the four Polya principles and how to apply them. I could ask you about any of the problem solving methods, but you should pay attention to the more difficult strategies: pattern recognition, using algebra, and the pigeonhole principle.
- From 2.1: Know the definitions from set theory. (See review for exam 1).
- From 2.2,2.3: Be familiar with the various models for addition and subtraction of whole numbers. Know about Venn Diagrams.
- From 2.4: Be familiar with the various models for division and multiplication of whole numbers.
- From 3.3: Know how to perform the various algorithms for adding and subtracting. In particular, be very comfortable with idea of exchanges in addition and subtraction. Be able to use place value cards, diagrams, instructional algorithms, and the final algorithms.
- From 3.4: Know how to perform the algorithms for multiplication and division. Be able to use place value cards, diagrams, instructional algorithms, and the final algorithms.
- From 4.1: Know all the definitions from 4.1.
- From 4.2: Know the tests for divisibility (except for 7,11,13).
- From 4.3: Know how to compute the GCD and LCM using *both* set intersections and prime factorizations.
- From 5.1-5.3: Know how to use the various models of integers to solve arithmetic problems.
- From 6.1: Know the basic models for fractions.
- From 6.2: Know the models for fraction addition.
- From 6.3: Know how to multiply and divide fractions.
- From 6.4: Understand the properties of rational numbers. Understand how rational numbers are ordered and be familiar with the density property.

Example Problems:

1. List the first 6 even and odd whole numbers. Come up with a formula for the n-th even number and n-th odd number. Based on your answer, every even number and every odd number has a particular form. Use these forms to prove that the product of an even and odd number is an even number and the product of an odd number and an odd number is odd.

2. All problems from exams I-III (except for problems on material not included in the topics list).

3. Find a formula for the n-th term of the sequence $1, 4, 7, 10, 13, 16, ...$

4. Your friend is worried about her grade in Economics 201. Since you are in MA 201 she hopes you can tell her what grade she needs on the final to keep get a B average (80) or above. Her first 3 exam scores were 70,65,and 75. Assuming her grade is the average on the 3 exams and the final (and each have equal weight), what do you tell her?

5. (a) Explain the pigeon hole principle.
   (b) There are 10 math books and 10 education books in a box. If you are blindfolded, what's the smallest number of books you could take from the box and be sure that you had at least one of each type?

6. (a) Given the sets A and B, provide both a verbal and a mathematical description of the sets $A \cup B$, $A \cap B$, $\bar{A}$
   (b) Draw the Venn-diagram representation form $A \cup (B \cap C)$ and $(A \cap B) \cup C$.
   (c) Draw Venn-diagram representations for (a) two sets which are disjoint and (b) two sets whose intersection is not $\emptyset$.
   (d) Problem 3 on page 87.
   (e) Problem 14 on page 102.

7. (a) Explain the missing addend model for subtraction of integers.
   (b) Set up and solve a word problem using the missing addend model.
   (c) Explain the set model of addition.
   (d) Set up and solve a word problem using the set model of addition.
   (e) Explain the missing factor model of division.
   (f) Give an explanation for why division by zero is not allowed.

8. (a) Solve the problem 4×3 in the multiplication tree model. What would be a good word problem for this computation?
   (b) Solve the problem 3×3 in the Cartesian product model. What would be a good word problem for this computation?
   (c) Solve the problem $28 \div 7$ in the repeated subtraction model. What would be a good word problem for this computation?

9. (a) Explain the concept of exchanges in addition. Illustrate your answer with examples.
   (b) Use place value diagrams to compute $278 + 963$, $763 - 469$.
   (c) Use expanded notation to compute $22 \cdot 311$. Rework the problem with the instructional and then the final algorithm. Explain the relationship between the two methods in the context of this problem.

10. (a) What does it mean to say that b divides a? Give a mathematical and intuitive definition.
    (b) Suppose that b divides a and c divides b. Does c also divide a? Prove or disprove your answer.
    (c) Suppose that b divides $a \cdot c$. Must b divide one of a or b? Prove or disprove your answer.

11. Find the GCD and LCM of 450 and 360 using prime power representations. Find the GCD and LCM of 48 and 72 using the Euclidean Algorithm.

12. Explain why for any whole number $n > 0$: $GCD(n, n) = n$, $GCD(n, 0) = n$, $LCM(n, n) = n$, $LCM(n, 1) = n$.

13. Fraction arithmetic problems:
    (a) 6.3 problem 5, p. 387
    (b) 6.3 problem 6, p. 387
    (c) 6.4 problems 5,6, p. 399

14. (a) 6.3 problem 10, p. 400
    (b) 6.3 problem 14, p. 400
    (c) 6.3 problem 16, p. 400
    (d) Give a definition for the set of rational numbers.
    (e) Explain the density property of the rational numbers.
    (f) Is every number a rational number? If not, what's an example?
    (g) Let W be the set of whole numbers, $\mathbb{Z}$ be the set of integers, $\mathbb{N}$ the set of natural numbers, and $\mathbb{Q}$ the set of rational numbers. Put these sets in order using "$\subset$". Explain your answer.

15. True or false? Explain.
    (a) Every integer is a rational number.
    (b) $n(\emptyset) = 0$
    (c) $\emptyset = \{0\}$
    (d) Every natural number larger than 1 can be written as product of powers of prime numbers.

16. *Sets.*

    (a) Draw the Venn-diagram representation for $\bar{A} \cup (B \cap C)$.

    (b) There are 100 students in the sixth grade and there are three after school clubs: art, music, and sports. Suppose 60 students are in the art club, 30 are in the music club, and 30 are in the sports club. Suppose also that 20 students are in both art and music, 15 are in art and sports, and 10 are in both music and sports. There are 5 students in all three clubs. How many students are in more than one club? You must show your work.

17. *Patterns.*

    (a) Prove that the product of any two even numbers is even.

18. *Problem Solving.* Solve the problems below. Identify the problem solving method you are using.

    (a) Find a formula for the n-th term of the sequence $1, 6, 11, 16, 21, 26, 31, \ldots$.

    (b) Your friend is worried about her grade in Thermodynamical Systems 101. Since you are in MA 201 she hopes you can tell her what grade she needs on the final to get at least a C average (70 or above). Her first 3 exam scores were 70, 65, and 60. Assuming her grade is the average on the 3 exams and the final and that the exams and the final have equal weight, what do you tell her?

19. Write a paragraph (at least 4 sentences) explaining the concept of exchanges in addition. Illustrate your answer with diagrams, examples, etc.. Your answer will be graded on the quality, detail, and clarity of your explanation.

20. *Fraction arithmetic.* Compute the following and put your answer in simplest form. Show all your work.

    (a) $\frac{6}{11} \div \frac{4}{3}$

    (b) $2 \div 5\frac{1}{3}$

    (c) $\frac{2}{3} \times \left(\frac{3}{8} - \frac{5}{4}\right)$

    (d) $\left(\frac{3}{5} - \frac{3}{10}\right) \div \frac{6}{5}$

21. *Divisibility.*

    (a) What does it mean to say that b divides a? Give a mathematical and intuitive definition.

    (b) Prove that if b divides a and c divides b then c also divides a. (Hint: Use the mathematical definition of divisibility.)

    (c) Suppose that a divides b and a divides c. Prove that a divides $b + c$. (Hint: Use the mathematical definition of divisibility.)

22. *Number systems.*

    (a) Give a verbal and a mathematical definition for the set of all rational numbers.
    (b) Explain the density property of the rational numbers.
    (c) Find a rational number between the numbers $\frac{3}{5}$ and $\frac{7}{8}$.
    (d) Is every number a rational number? Explain.

23. Identify the following statements as true or false. *Explain your answer.*

    (a) Every integer is a rational number.
    (b) If $n(A) = 3$ and $n(B) = 5$ then $n(A \cup B) = n(A) + n(B) = 7$.
    (c) $\emptyset = \{0\}$.
    (d) 0 is neither even nor odd.
    (e) An even number times an even number is always odd.
    (f) The set $\{0, 1, 2\}$ is closed under multiplication.
    (g) For any nonzero whole number b, $\frac{b}{b} = 1$.
    (h) The set of prime numbers is finite.

24. Find three rational numbers between $\frac{1}{4}$ and $\frac{2}{5}$.

25. When asked to evaluate $\frac{1}{3} + \frac{3}{5}$ your student claims the answer, when simplified is $\frac{1}{2}$. What do you think the student did wrong? What would you say to help them understand?

26. Problem number 9 from page 399.

27. Perform $(-5) \times 3$ on a number-line.

28. Suppose that you know $a \to b$ is true. If b is false, what can you conclude?

29. Suppose that you know $a \to b$ is true. If a is true, what can you conclude?

30. Suppose that you know $a \to b$ is true. If b is true, what can you conclude?

31. If $a \to b$ is true, and a is false, is b false?

Study Tips

(a) Begin studying today!
(b) Glance through the book/notes and identify any topics that you do not know/understand. Read more about these, and ask me questions.
(c) Do the entire review sheet.
(d) Do the practice exam.
(e) Look over the homework. (All solutions are on the webpage.)
(f) Get together with other people and discuss the concepts.
(g) Get a good night's sleep the night before and relax the morning of the exam.

Good Luck!