#1 See book for divisibility tests.

If $c|a$ and $c|b$ then:

$a = ck \qquad b = cj$ for some $k, j \in \mathbb{N}$.

(a) Now, $a+b = ck+cj = c(k+j)$

and, $c \mid c(k+j)$ because

$c(k+j) = c * (k+j) + 0$. So, $c|a+b$.

(b) $a-b = ck-cj = c(k-j)$.
and $c \mid c(k-j)$ because

$c(k-j) = c*(k-j)+0$. So, $c|a-b$.

(c) $a \cdot b = (ck)(cj) = c^2(kj)$

and $c \mid c^2(kj)$ because

$c^2(kj) = c * c(kj) + 0$. So, $c|a \cdot b$.

#2 This is part (a) above.

#3 An even number can be written as $2k$. An odd number can be written as $2j+1$ where $k, j \in \mathbb{N}$.

So, the product of our two numbers is $2k(2j+1) = 4kj + 2k$

$= 2(2kj+k)$ so $2 \mid 2(2kj+k)$

because $2(2kj+k) = 2 * (2kj+k) + 0$

so, $2 \mid 2k(2j+1)$ which means the product of our two numbers (an even and an odd) is even.

#4 $36,\underbrace{335},936,\underbrace{637}$ div. by 7, 13, 11? ← grouped into alternating sets of three digits.

$$\begin{array}{r} 637 \\ +335 \\ \hline 972 \end{array} \qquad \begin{array}{r} 936 \\ +36 \\ \hline 972 \end{array} \qquad \begin{array}{r} 972 \\ -972 \\ \hline 0 \end{array}$$

added the two groups

← subtracted the sums.

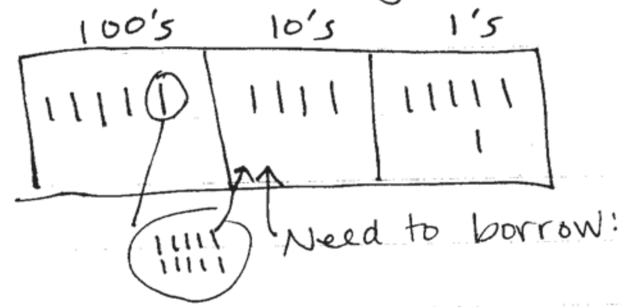
And $7 \mid 0$, $11 \mid 0$, $13 \mid 0$ So, our number is divisible by 7, 11 and 13.

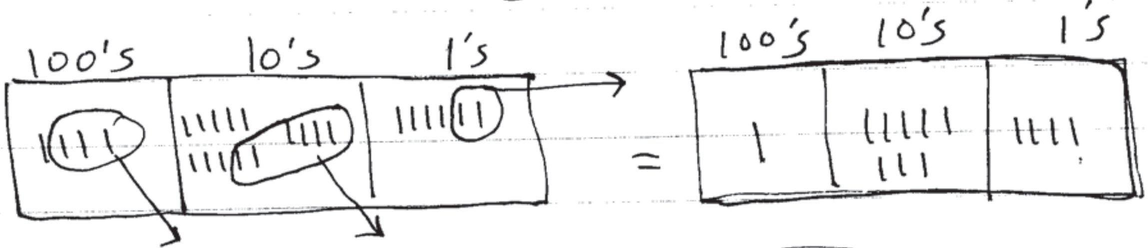
"7 divides zero"

#5 (a)

Place-Value Diagram

$$\begin{array}{r} 546 \\ -\,362 \\ \hline \end{array}$$



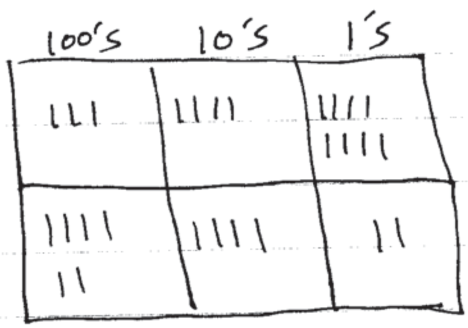


= 184

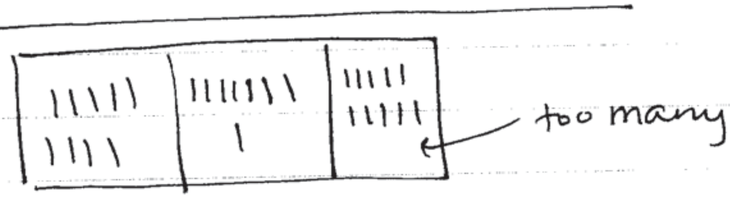
Check:

$$\begin{array}{r} \overset{4}{\cancel{5}}46 \\ 362 \\ \hline 184 \end{array}$$ ✓

(b)



$$\begin{array}{r} 348 \\ +\,642 \\ \hline \end{array}$$



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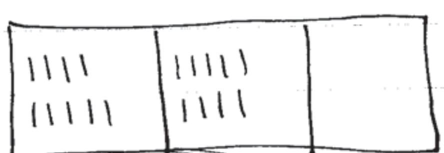


= 990

Check:

$$\begin{array}{r} 3\overset{1}{4}8 \\ +\,642 \\ \hline 990 \end{array}$$ ✓

#6 $444_{ten} \longrightarrow$ base five.

$$\begin{array}{r} 3 \\ 125\overline{)444} \\ \underline{375} \\ 69 \end{array} \qquad \begin{array}{r} 2 \\ 25\overline{)69} \\ \underline{50} \\ 19 \end{array} \qquad \begin{array}{r} 3 \\ 5\overline{)19} \\ \underline{15} \\ 4 \end{array} \qquad \begin{array}{r} 4 \\ 1\overline{)4} \\ \underline{4} \\ 0 \end{array}$$

$$444_{ten} = \underline{3} \times 5^3 + \underline{2} \times 5^2 + \underline{3} \times 5^1 + \underline{4} \times 5^0$$

$$= 3,234_{five}$$

#7 $321_{five} \longrightarrow$ base ten

$$3 \times 5^2 + 2 \times 5^1 + 1 \times 5^0 = 3 \times 25 + 2 \times 5 + 1 \times 5^0$$
$$= 75 + 10 + 1 = 86_{ten}$$

#8 $\begin{array}{r} 132_{five} \\ + 221_{five} \\ \hline \end{array}$

	25's	5's	1's
	I	III	II
+	II	II	I

=	III	IIIII	III

too many

=	IIII		III

$= 403_{five}$

#9

$$
\begin{array}{r}
451 \\
\hline
1 \\
50 \\
+\ 400 \\
\hline
12 \,\big)\, 4\cancel{5}\,{}^{1}4\,1\,3 \\
-\ 4\,8\,0\,0 \\
\hline
6\,1\,3 \\
-\ 6\,0\,0 \\
\hline
1\,3 \\
-1\,2 \\
\hline
1
\end{array}
\quad = 451\,R\,1
$$

#10 ⓐ gcd(a,b) is the largest whole number m that divides both a and b.

ⓑ lcm(a,b) is the least whole number m that is a multple of both a and b.

ⓒ gcd (1092, 525).

$1092 = 2^2 \cdot 3^1 \cdot 7^1 \cdot 13^1$

1092 → 2, 546
546 → 3, 182
182 → 2, 91
91 → 7, 13

$525 = 3^1 \cdot 5^2 \cdot 7^1$

525 → 5, 105
105 → 5, 21
21 → 3, 7

$\gcd(1092, 525) = 2^0 \cdot 3^1 \cdot 5^0 \cdot 7^1 \cdot 13^0$
$= 21$

(d) $\text{lcm}(1092, 525) = 2^2 \cdot 3^1 \cdot 5^2 \cdot 7^1 \cdot 13^1$
$= 27,300$

or: $\text{lcm}(1092, 525) = \dfrac{1092 \times 525}{\gcd(1092, 525)}$

$= \dfrac{573,300}{21} = 27,300$

#11 $n = a \cdot b$ is it true that if

$c \mid n$ then $c \mid a$ or $c \mid b$?

No! Consider $n = 12$, $a = 3$, $b = 4$, $c = 6$.

It is clear that $6 \mid 12$ and $3 \cdot 4 = 12$.

However, $6 \nmid 3$ and $6 \nmid 4$.

#12 abc, $\underbrace{abc}$ ← group into sets of three digits

abc abc subtract the sums: $\begin{array}{r} abc \\ -abc \\ \hline 0 \end{array}$
find sums

$7 \mid 0$, $11 \mid 0$ and $13 \mid 0$.
So, 7, 11 and 13 divide abc,abc.

#13 (a) Find the missing whole number.

$x \div 5 = 7R1$

$$\begin{array}{r} 7 \\ 5\overline{)\,x} \\ -35 \\ \hline 1 \end{array}$$

So, $x - 35 = 1 \Rightarrow x = 36$

Check: $36 \div 5$: $\begin{array}{r} 7 \\ 5\overline{)\,36} \\ -35 \\ \hline 1 \end{array} = 7R1$ ✓

(b) $47 \div y = 4R3$

$$\begin{array}{r} 4 \\ y\overline{)\,47} \\ -(4y) \\ \hline 3 \end{array}$$

So, $47 - (4y) = 3$

$\Rightarrow 44 = 4y$

$\Rightarrow 11 = y$

Check $47 \div 11$: $\begin{array}{r} 4 \\ 11\overline{)\,47} \\ 44 \\ \hline 3 \end{array} = 4R3$ ✓

#14 a.) No! FALSE.

1 is a whole number, and 1 can't be written as a prod. of primes because 1 = 1 is not prime.

b.) False. For example $2 \times 4 = 8$ is not odd.

(c) False. $\text{lcm}(5,10) = 10$, but $5 \times 10 = 50$.
So, you can't just multiply the #s together.

(d) TRUE.

(e) False. For example $2 \times 2 = 4$ and
$4 \notin \{0, 1, 2\}$.

(f) No! You can't divide any number by zero.
If $a \div 0 = 0$ then $a = 0 \times 0$
$\Rightarrow a = 0$, but not every whole number
is 0. So, this statement was False.

(g) TRUE.

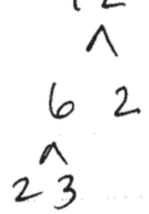
(h) False. We proved there are infinitely
many by taking products of primes
and adding 1 to see that # is also prime.

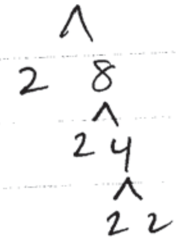
(i) FALSE. 0 doesn't divide any number.
If $0 \mid 0$, then $0 = 0 \cdot k$ for one
unique $k \in \mathbb{N}$. However, any number
works for k, so there is no unique one.
This means $0 \nmid 0$.

(j) ~~TRUE~~. We showed this.

(#15) We just need to find the largest whole # factor of 12 and 16. This means we want to find gcd (12, 16).

$12 = 2^2 \cdot 3^1$ $\qquad$ $16 = 2^4$

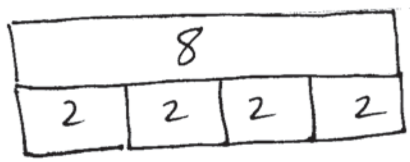




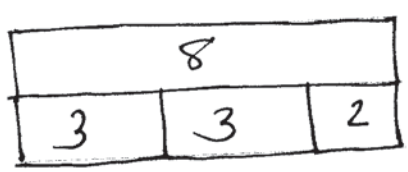
$\gcd(12,16) = 2^2 \cdot 3^0 = 4$

So, he will use 4ft x 4ft tiles.

(#16)

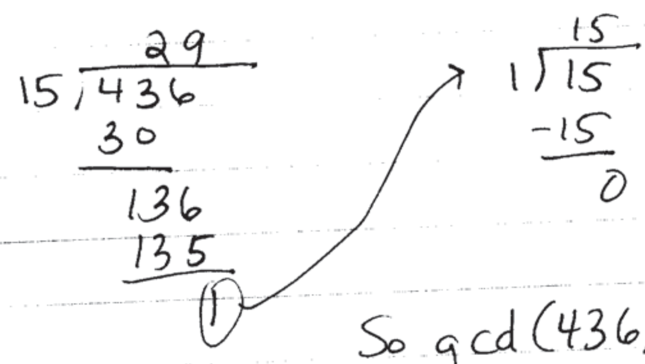


shows $8 \div 2 = 4R0$



Shows $8 \div 3 = 2R2$

(#17)



So $\gcd(436, 15) = 1$ because it is the last nonzero remainder.

$$\operatorname{lcm}(436,15) = \frac{436 \times 15}{\gcd(436,15)} = \frac{6540}{1} = 6{,}540$$

#18 "0" (zero)
This is the only correct answer because to be divisible by ten, the number's last digit must be 0.

#19 $\sqrt{317} \approx 17.8$ So, we check to see if any prime $p \leq 17.8$ divides 317.

$2 \nmid 317$, $3 \nmid 317$, $4 \nmid 317$, $5 \nmid 317$, $7 \nmid 317$, $11 \nmid 317$, $13 \nmid 317$ and $17 \nmid 317$. This means 317 is prime.

#20 Notice that $3 \mid 327$ because $3+2+7=12$ and $3 \mid 12$.
So, 327 is composite.

#21 This is shown in the book and we did this twice in class.

#22 My opinion is yes because students won't be able to multiply multi-digit numbers if they don't already know "easy" multiplications and still rely on manipulatives and the array models. However, students should be introduced to multiplication as repeated addition and be comfortable with the concept before being asked to memorize.

#23 $24 = 2^3 \cdot 3^1$.

$$24 \to 2,\ 12;\quad 12 \to 2,\ 6;\quad 6 \to 2,\ 3$$

There are $(3+1)(1+1) = 4 \times 2 = 8$ factors.

They are: $\{1, 2, 3, 4, 6, 8, 12, 24\}$.

$$\begin{array}{ll} 3^0 = 1 & 3^1 = 3 \\ 3^0 \cdot 2^1 = 2 & 3^1 \cdot 2^1 = 6 \\ 3^0 \cdot 2^2 = 4 & 3^1 \cdot 2^2 = 12 \\ 3^0 \cdot 2^3 = 8 & 3^1 \cdot 2^3 = 24 \end{array}$$

#24 $68 = 2^2 \cdot 17^1$ So, there are

$$68 \to 2,\ 34;\quad 34 \to 2,\ 17$$

$(2+1)(1+1) = 3 \times 2 = 6$ factors.