Orientation for Teaching MA 202

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Contents

1 Instructing Philosophy

Welcome to teaching "Math for Future Elementary Teachers: Part II"! Throughout this orientation packet, we describe how the course is structured and what to expect from you (as an instructor) and the students. So with that, what is the point of this course? Why are we here? As in other classes, we are here to help our students (pre-service teachers [1]) with their conceptual understanding of the content of mathematics. Please note that there will be a pedagogy (or "methods") course taught in their junior year, so your focus should be on the content. Also, even if you've taught MA 201 previously, you will need the material in this packet, as the PST's will have a better idea of what we're trying to do. Thus we need to challenge them even further.

When a student asks a question or comes up with a procedure, we challenge the PST to determine whether it correct or incorrect. In doing so, we, as instructors, will try to help the PST deepen their understanding of the mathematics of elementary school. We also want to develop a mathematical habit of the mind. An example is given below, in which a student has performed the operation $70 - 47$:

$$\begin{array}{r} 70 \\ -47 \\ \hline 3-3 \end{array} \longrightarrow = 27$$

As you can see, the algorithm given gives an incorrect answer. We want the PST's to understand why this is incorrect using a conceptual explanation.

Before we get into more specifics, take some time to come up with a definition of conceptual understanding ...

Now take some time to read the attached article by Thompson[2], Philip, and Boyd. This article will be discussed during the orientation.

It is essential, as an instructor, to have a working definition. Past experience has shown that PST's can provide somewhat proficient definitions of conceptual understanding. But when asked to provide examples of conceptual problems, more lacked proficiency. Even after they've had MA 201, some PST's still have a rough definition of conceptual understanding. This typically was not the case with procedural understanding, where PST's showed a better understanding. Thus we want to focus more on the conceptual understanding in this course, because without true understanding, all is rote and nothing is of depth.

As an exercise, consider these 4 sample questions below (with a focus on the MA 202 material). On a scale from 1-5, where 5 represents the question being very conceptual in nature, how would you rate these questions?

1. An ordinary deck of playing cards is shuffled and a card is drawn at random. What is the probability that the card is a spade or a face card?

2. How many different three letter words can be made using the letters a,b,c,d,e, where you can not have repetitive letters?

3. A crew boat can travel speeds around 15 mph. If the oars stay out of the water for 1 second, and the boat speed is constant, how far will the boat go when the oars are out of the water?

[1] PST := Pre-Service Teacher

4. The average height for males is 6 feet. Which of the following best describes the standard deviation : 5 in, 10 in, or 15 in? Justify your reasoning.

We hope that most would agree on how to rate these, but this is not always the case. Furthermore, how could we change the questions to be more conceptual based?

2 Student and Class Expectations

Now that we have a global philosophy in place, let's talk about expectations. As noted before, your students will have a rough conceptually thought out idea of what conceptual understanding is (from MA 201). But, as examples show, their interpretation still lacks understanding in some way. For example, when asked : "How do you define conceptual knowledge and give an example?", past (MA 202) responses were:

- It is knowing the proper way to complete a problem; e.g. knowing about functions and how they work.
- It is the knowledge of concepts or ideas (behind the procedures); e.g. the Pythagorean Theorem.
- It is the ability to apply information in a meaningful situation; e.g. knowing what a derivative is and what it represents.
- It is learning concepts to do math; e.g. probability.

You should have class discussions about conceptual knowledge and examples (throughout the semester), so as to help the PST's understand even better.

Next, a discussion should be had as to what sorts of standards we (as math teachers) need to follow when teaching math at the K-6 level (the classes primary focus). As they've already completed the first course of this sequence, the PST's should have a good idea of this. Your discussion will lead to the *principles and standards*. The *principles and standards*[2] were set by the *National Council of Teachers of Mathematics* (NCTM) in a document called **Principles and Standards for School Mathematics**. This document was published in 2000, and has been approved by many of the professional societies. Kentucky and Tennessee (as well as most other states) have their teaching standards aligned with these. The following definitions come from the reference at the end of this section.

Definition 2.1 *A principle is a statement which reflects basic precepts that are fundamental to a high quality mathematics education. There are 6 of them, and we will engage in classroom discussion to get the students to give as many as possible. The principles are*

- Equity – High expectations and strong support for all students
- Curriculum – A coherent, focused on important mathematics, and well articulated across grade levels
- Teaching – Requires understanding of what students know and need to learn and then challenging and supporting them to learn it well
- Learning – Students must learn with understanding, building new knowledge from experience and prior knowledge
- Assessment – Should support the learning of important math and furnish useful info to both teachers and students
- Technology – It is essential in teaching and learning and influences how math is taught

[2]For those who haven't taught this class before, take time to read the following. You should remind the students what the principles and standards are, but lengthy discussion is reserved for the first course. Thus you shouldn't need to spend much time with the class on this.

Definition 2.2 *A standard is a description of what mathematics instruction should enable students to know and do ; i.e. what is valued for school mathematics education. The first five describe mathematical content goals and the last five address processes. They are:*

CONTENT STANDARDS

- Numbers and Operations – includes estimation
- Algebra – includes $x + 7 = 11$
- Geometry
- Measurement – includes counting, estimating, and formulas
- Data Analysis and Probability – includes mean, median, and mode

PROCESS STANDARDS

- Problem Solving
- Reasoning and Proof
- Communication – to peers, teachers, and others, analyze and evaluate the math thinking and strategies of others, use math ideas precisely
- Connections – between math ideas and to other fields
- Representation – modeling among other things

In teaching this course, emphasis is also put on getting the PST's to comprehend these guidelines, as when they become teachers, they will be expected to know and follow them. Ultimately, though, focus should be on making sure the PST's know the **math content**. In MA 201 and 202, you will NOT be teaching the PST's how to teach, as they will do this in a future course. To achieve content knowledge in a conceptual way, we strongly urge that you have the PST's work in groups to share their ideas with each other (i.e. don't just lecture the whole semester). To easily remember the principles and standards, just remember ECTLAT and $5 + 5$. Note that all of these are in the back cover of the book.

Reference: The NCTM website is www.nctm.org

3 Book and Chapter Description

In this section, we provide a description of the chapters covered in the textbook, as well as sample homework problems and exam layout. First and foremost, the textbook for the course is *Mathematical Reasoning for Elementary Teachers* , written by Calvin Long and Duane DeTemple, **fourth edition**. In MA 201, Chapters 1 through 7 are covered, and Chapters 8 through 13 are covered in MA 202. Note that the only chapter that is not covered is Chapter 5, which the students covered in doing the presentations in MA 201. Unlike MA 201, the chapters in MA 202 should be covered in order from Chapter 8 to Chapter 13. As you will see, the latter half of the book covers Algebra, basic statistics and probability, basic geometry, and transformations. Most PST's who have taken MA 201 find that the chapters covered in MA 202 are more challenging. Please take time to read through the chapter descriptions provided later in this section. Provided after the chapter description is a list of suggested homework problems. You do not need to assign these exactly, but you should look over these problems to get a good idea of questions for exams.

Exams in this course will have a different approach than other courses you've taught. Throughout an exam, emphasis should be put on conceptual learning and the mathematics content understanding needed to teach in elementary and middle schools. Students should be able to formally define a term , and also say what the definition means, what they "feel like" (an intuitive understanding), and to picture them. While there will be some questions which involve procedural skills, the majority of the questions should be like the following. Many of these questions deal with "pedagogical content knowledge", with the exception of (H). Questions do not need to be restricted to these, but they are great guidelines.

(A) Is this procedure (statement) correct? Why or why not?

(B) Students A and B propose the following as true (or do a procedure). Who is correct, A or B, and why?

(C) Why does the following procedure work?

(D) Is it true that ...? If the student answers yes, then they should provide a proof or convincing argument. If the student answers no, they need to give a counterexample. Note that frequently students will answer yes, but then provide a specific example as a "proof".

(E) Make up a problem that gives an example of the following operation or ....

(F) Apply a concept to a various situation. (Elementary schools are big on applications and connections these days ... which is a good thing)

(G) How would you explain the concept of ???? to an n^{th} grader? (can substitute literate adult, mathematician, present teacher, other audience for n^{th} grader)

(H) Demonstrate you know the skills of the material covered. Do procedures to show you can compute basic problems or find basic properties.

DO NOT DUPLICATE

CHAPTER EIGHT (*Algebra*)

This chapter is (or should be) review, as MA 109 is required for this course. Using algebraic expressions and recognizing that some graphs are straight lines is important, but there are lots of non-straight graphs which are just as important. Don't spend too much time on the various forms of the equation of a straight line ; only the slope intercept form is necessary for future text purposes. The important concept in that section is the notion of slope and the distance formula (p. 498), of which they should be fluent after you've discussed them. If they are not, either work with the students yourself or send them to Mathskeller. It is suggested to skip most of p. 507-511, with the exception of slope-intercept form. All sections are covered in this chapter (with the noted exceptions).

DO NOT DUPLICATE

CHAPTER NINE (*Statistics*)

This is a rather easy chapter, and one which the students should have no trouble with. All sections should be covered in Chapter 9. They should be able to read the text easily, and you should feel free to suggest that and skip much of the verbiage of Section 9.1. What they will miss on reading the text is the big picture, mainly: "How does one describe data?" Philosophically, our point is that the more data points shown, the more difficult it is to figure out what it all means. The fewer data points we have, then we have more information hidden.

An informed citizen needs to understand the complexities of statistics. It is a very complicated subject, and one needs to read data and the conclusions from that data carefully. Here is a (real) example : An educational researcher did a study at a technological university which computed the grade point average of men and women upon graduation. Since the women's GPA was higher , the researcher asserted that women succeeded better at this institution. There were many difficulties in this situation. One is in the assumption that all students had the same ability on entering, and so were equally likely to do well. If ability was measured by ACT or SAT scores (a logical assertion), this shows an invalid conclusion, as women's GPA's for each entering class was higher than the men's. Also, what do you do about people who drop out? There are many variables that were not discussed, and so the result does not hold its weight.

The point of the chapter, then, is to get the first descriptors of a data set. Simply put, how "wide" is the data? How many times does one datum point repeat (frequency)? Where is the middle (typically) and what does it mean to be the middle (mean v. median)? What is the spread of the data set (range, variance, standard deviation)? The five-number summary (p. 554) is important, but it is crucial to explain that even this collection of numbers does not describe the set in detail. An quote that sums up this subject goes as follows: "A fellow with his head in the sauna and his feet in the snow will feel pretty good, on average." Problem 7 on p. 561 is a nice exercise to show why the variance or standard deviation needs to be used.

For Section 9.3, you will need to teach the students how to use a calculator to compute variance, standard deviation, and mean. Check with the class sufficiently far in advance to see what calculators they have. Should they prefer to do this process with computer software, that is fine . . . but it is your decision. Ultimately, though, they should understand the ideas of mean and variance in a general setting. Discussions of Z-scores should be held to a minimum.

DO NOT DUPLICATE

CHAPTER TEN (*Probability*)

The first of our challenges in this section (which should be the hardest of the chapters covered in MA 202) is to convince the students that they really need the material. They need to know that probability is covered in elementary school, both for its own sake and to give a very important use to fractions. Make sure that the students read the box on p. 591-592, which gives the NCTM standards for probability in grades 3-5. Probability is also in the Kentucky standards.

The property on p. 595 is quite important, and some time should be spent on it. Emphasize to the class that if the events are mutually exclusive, the problem reduces to addition of probability of each of the two sets. Empirical probability is something that is done frequently (no pun intended) in elementary school, so please cover example 10.8 on p. 598-599. You should assign problems similar to this one.

Combinations and permutations are the heart of the mathematics, and a difficulty for them in this chapter. The difference between ordered and unordered selections is very hard for students at this level. Just the factorial notation and symbols has caused fear in the hearts of past students. The explanation of the difference between permutation and combination is not at all good (p. 615-616), and you will need to supplement it with many examples. One might even try to do the material without the formulas at the bottom of p. 616. Be prepared to spend much time (and two problem sets) going over the material of Section 10.2.

To do Section 10.3 "right" (that is to say with depth – you will all present it correctly!) would require more than two weeks. Again, having two problem sets over this material will be necessary, so that the students have a chance to let the material sink in. We have allocated much time to these three sections for this very reason (which is all of Chapter 10). You might choose to omit conditional probability, which is fine. (Please let Dr. Millman[3] know what you are doing, though.)

[3]Let's talk about how these 3 chapters are going as you get further into the material. I am especially interested in the timing and depth that you will achieve. It is better that they understand the material at an appropriate (for a future elementary teacher) level than to go over too much too fast. Note that this is the criticism of the elementary school curriculum – broad but too shallow!

DO NOT DUPLICATE

CHAPTER ELEVEN (*Geometric Figures*)

This chapter starts off by talking about figures in the plane. Past experience has shown that the PST's understand the ideas of this section, but struggle with the definitions and terminology. At any rate, there should not be much time spent on Section 11.1.

Section 11.2 deals with curves and polygons in the plane. What you will notice is that this section is quite lengthy, and as such you should spend more than one day on it. One of the first ideas that comes up in this section is the Jordan Curve Theorem. Although this is interesting mathematics, you should think of cutting this out of your lesson as a time saver. Further in the chapter, the classification of quadrilaterals is discussed with a Venn diagram given. Some discussion should be had concerning this diagram, and how to interpret it. Other theorems in this section provide a great opportunity to get the students to practice their conceptual interpretations of the material.

Figures in space and networks are the last two topics of the chapter. To aid in teaching figures in space, do not forget about the manipulative kit and the geometric figures. Having the students physically feel the figures helps them in understanding the differences between figures. Further activities, such as building figures based on cutouts, would also enhance the PST's understanding. The other topic concerning networks should not be covered due to time constraints.

If you did not cover Section 11.3 before exam 2, or would like to revisit it in more detail (for example, nets are important), it is one of the possible presentation topics. That would be a good way to cover the section and preserve time.

DO NOT DUPLICATE

CHAPTER TWELVE (*Measurement*)

With the exception of tangrams, this chapter is (or should be) review. On the other hand, the presentation of unit analysis (p. 752) is very helpful in remembering "which way to convert". Do not get bogged down in the various units presented. You are welcome to omit weight and mass. The definition of "congruence" (p. 757) may offend you, but remember this is an informal approach, and the level of abstraction that would be needed to do it right is well beyond this course. Leonardo's problem is a very clever use of the techniques of this section. Deriving the area of triangles, trapezoids, and others is quite relevant. Elementary teachers spend a lot of their mathematical life working with such things. Using a geoboard (look in the AMSP kit) is a great idea. Deriving the area of a circle, as done in the book (or some other informal way), is needed. If not, then we are just teaching them procedures without concepts. Example 12.17 is a good in-class problem for discussion, as it is very different from the usual Pythagorean problems. At this point, remind Dr. Millman to place two "proofs without words" of the Pythagorean theorem in your mailbox. Should anyone want a laugh (or maybe not...), Dr. Millman has a truly horrendous pun on the Pythagorean Theorem that he would love to share!

The volume and surface area of a sphere, pyramid, and cylinder should be formulas that they retain. The "proof by picture" of these solids needs to be discussed. Perhaps you should implement the "rice experiment" in a class! For example : take a sphere of radius r and fill it with rice – then dump the rice out into a cube. How big does the cube need to be? Well, I hope you know this, but to be complete it is a cube of side length $\left(\sqrt[3]{\frac{4}{3}\pi}\right) r$. Assign it as a homework problem if you choose not to do this in class. You might also try to do the Power Solids experiments. Note: Dr. Millman[4] has a key to the closet in Mathskeller, which has the kits – feel free to come get the key.

In addition to all of this, ask your students to spend at least 15 minutes at the web site given below. It is absolutely full of delightful manipulatives. One way to do this is to assign it as a homework, where the students must pick an applet from this sight and write about it. The site is:

National Library of Virtual Manipulatives for Interactive Mathematics

http://matti.usu.edu/nlvm/nav/vlibrary.html

[4]I continue to be especially interested in the timing and depth that you will achieve. In years past, 202 only covered through Chapter 12. It is better that your students understand the material at an appropriate (for a future elementary teacher) level than to go over too much too fast. Note that this is the criticism of the elementary school curriculum – broad but too shallow! Yes ... this is the same note you saw before. This is the motivating factor in ly suggestions for Chapter 13

DO NOT DUPLICATE

CHAPTER THIRTEEN (*Transformations, Symmetries, and Tilings*)

Instead of covering Sections 13.1 and 13.2 (rigid motions and similarity transformations), one of the presentations (see section 4 of this orientation packet) assigns "Three Types of Symmetry". This is investigation 1 of CONNECTED MATH, "Kaleidoscopes, Hubcaps, and Mirrors", grade 8 (Geometry). It is recommended that you require this presentation for your students who are going into middle school teaching. Typically there are 2-5 middle school teachers, and this works out best. The material is also covered in Section 13.2 of the textbook, but the CONNECTED MATH text is easier to follow. Be sure to tell the class that this material will be on the final exam. You can "clean up the material" after all the presentations are made. In addition, you will cover tiling (Section 13.3). Section 13.3 is independent of the first two sections. Tilings (or tessellations) and symmetries are done in elementary school. For example, see Unit 10 "Reflections and Symmetry" of EVERYDAY MATH, 5^{th} grade.

Suggested Homework Problems

In general, let's try to assign problems from the book each week which require a "written" (as opposed to numerical) answer. It is also important to note that the state assessments (and state standards) are aligned with the NCTM standards. Thus at least one problem each week should come from "state assessments" problems, such as exercise 30 on p. 41. For the ease in coordination, you should assign whatever blue-type problems (whose answers are in the student manual or back of the book), and then have some black-type which can be used for grading. Of course, you ultimately get to decide what problems to assign. Also, make sure that you personally write out solutions to the problems , so that you don't get caught off guard. This is a good rule of thumb for any class, but is still worth noting! Note that the homework problems come from the **fourth** edition of the textbook. Some of your students might have the third edition, so please make a note of this to your class. The problems are broken up into suggested sets. Now onto the suggested problems:

Problem Set	Section	Problems
I	8.1	3,8,9,13,15,18,20(to challenge them),28,39
	8.2	1,3,10,11,12(could be in-class group assignment),18a,36
II	8.3	2,6abc,7,9,13a,17ab,26,32,38
III	9.1	1abc,4,6,7,12,17,18,25,32
	9.2	2acd,3,6bcde,7
IV	9.2	9,10ab,11c,14,15,17,19a,29,34
	9.3	1bd,2,8,11,17,18,20,31,33
	10.1	2,5 and 9(could be in-class group assignment),15a,17,18,20,29,34
V	10.2	3,4,6,9,15,17,19,21,29,35,37,41
VI	10.3	2,5,7abc,10,11,12,17,18,20,25,35,37
	11.1	3,5,12,13,14,17,19,23,29,42
VII	11.2	2,3,6,7,11a,13,18,20,26,41
	11.3	1,2,7,12,14,18c,23,26a,33
	12.1	4,5,7,11,12,20,21,29,31,34
VIII	12.2	1,6,7,9,10,13c,15,17,19,20,25,27,28,42,52
IX	12.3	2b,4c,6,9,10,15,22,28,32,40
	12.4	1,2d,3d,4b,5c,6c,9,11,16,19,22
	13.1	1,4,6,10,17,19,25,30,37,44
X	13.2	2,3,12,20,43
	13.3	7,8,27

NOTE: Some of the homework problems will be covered in the presentations. Be sure you read through the presentations before you assign problems which are not on this list.

4 Presentations

After spending most of the semester focusing on conceptual understanding and developing a mathematical habit of the mind, an opportunity is given to the PST's to demonstrate their conceptual understanding to you. Each PST in the class is expected to give a presentation on a pre-determined mathematical subject. Moreover, each PST will be a part of a larger group in giving said presentation to the class. The goals of this project are based on the content knowledge of the mathematics and on the NCTM standards. The goals of the project are

- For PST's to begin to learn to work collaboratively in groups
- For each PST in MA 202 to make an oral presentation about some part of mathematics
- For PST's to think about mathematics both independently and as part of a group
- For PST's to write about mathematics
- To acquaint the PST's with some journals and articles in the literature which is mathematics content based
- For PST's to present ideas with a conceptual emphasis.

As mentioned earlier, the PST's will be breaking up into groups. In MA 202, unlike MA 201, the group topics are more independent from the book (see next page for an in-depth description). Since there are a variety of topics to choose from, it is recommended that you let the PST's decide which topic they want to present on. In the past, having groups of no more than 4 worked out the best. After the topic is chosen, the group must then structure a presentation to explain the conceptual mathematics of the material to the MA 202 class. Note that the MA 202 class IS the audience for the presentation, as opposed to an elementary school class. *Each member of the group is expected to speak for* ***7-10 minutes***. The presentations will be evaluated (by you and another peer – see below) on these 4 criterion:

1)	Quality of the oral presentation as a presentation:	20 pts
2)	Preciseness and mathematical correctness:	20 pts
3)	Applicability in the classroom – did you relate the material to what you would expect to see in K-6?:	20 pts
4)	Creativity – did you bring something to the presentation other than what was in the book?:	10 pts

Peer Mentor Assessment Assignment: In addition to a presentation, each PST will be assessing another PST based on the 4 criterion listed previously. Each person (A) will be paired up with another person from another group (B) BEFORE the presentations begin. Person A will be asked to review person B's presentation *using the criterion* . Person A will then write a 1-2 page paper of their assessment of person B, and turn in the paper after the presentations are done. Note that the assessment will not affect person B's presentation grade. Person A's assessment of person B will be grade out of **30 points**, and added onto the project grade. Honesty, respect, and constructive criticism are a few expectations of the PST's in writing about a fellow classmate.

Student Reactions to the Presentations: After the presentations are completed, the PST's should fill out the questionnaire concerning the presentations in MA 201 and 202 (see disc for file). This information should be given to Dr. Millman. The information will be taken into consideration in terms of any changes that need to be made to the presentation aspect of the course. You (the instructor) do not need to grade this assignment. However, to ensure that the students give us useful feedback, you should consider giving them a score for turning it in. Moreover, since the presentations are so near the end of the semester, you should encourage them to turn in this questionnaire at the final exam as well.

Presentation Topics [5] for MA 202, Spring 2005

1. **Understanding Polygons** (Grade level 6, with applications to earlier grades): "Building Polygons" , Investigation 2 of CONNECTED MATH, grade 6 (Geometry), "Shapes and Designs". The presentation should solve the problems 2.1, 2.2, and 2.3. You may need to construct *polystrips*.

   Additional Comments: Constructing polystrips will be a great way to incorporate the class into the presentation, but make sure the strips are ready beforehand. Beware of the PST's classifying properties of different polygons, as this will have been done by you in lecture. Instead, the focus should be on how someone can conceptually see polygons, and distinguish the changes in polygons when items are changed (side length, angles, etc). The PST's should explain how each of these concepts are better seen using ideas like polystrips. This project should not turn into making different shapes . . . there should be substance to the presentation.

2. **" Polygon Properties and Tiling"**, Investigation 4 of CONNECTED MATH, "Shapes and Designs" grade 6 (Geometry). Make sure that you do problems 4.2 and applications, connections, and extensions (p. 47-49).

   Additional Comments: Just like the previous topic, this is another where the PST's will present material that has already been covered. The PST's will also focus on computations (measuring the angles), which should take up only a minute or two. Instead, the focus should be on having the PST's explain why the approach of measuring angles is useful in seeing a pattern for regular polygon. Challenge the PST's to investigate different ways to verify the formula for the sum of the interior angles of a polygon.

3. $*$ **Symmetries** (Grades 4-8): See "Three Types of Symmetry". Investigation 1 of CONNECTED MATH, "Kaleidoscopes, Hubcaps, and Mirrors", grade 8 (Geometry). **This presentation is required for those going into middle school teaching**. This material is also covered in Section 13.2 of Long and DeTemple. You should follow the CONNECTED MATH text, but feel free to integrate the Long and DeTemple material. Cover as much of the investigation as you can in your time. Be sure to do problem 1.2 (p.9) and translational symmetry (p.12) including problem 1.4, and some of problems 4,5,6, or 7.

   Additional Comments: As noted, this material is also covered in the book, so plan your lectures accordingly. Make sure the PST's ad there own wording to this topic. A suggestion for this topic is to bring in real-world items which represent a type of symmetry. The PST's should also discuss different types of shapes that do and do not create tessellations. Again, challenge them to go above and beyond what is given in the article and the book.

4. **Use of Art for Mathematics Education** (Grade level K-5): See the article by Richard Millman and Ramona Speranza, "Artist's View of Points and Lines", Mathematics Teacher Vol. 84 (1991), p. 132-138. What mathematical concepts can you present to your students by using works of art? Remember that sculpture can give 3-D intuition, for example, even though it is not in the article. This is not about the

[5] The added opinions are to give you a heads up on what the students might do. The student version of the presentation should not have the "additional comments" sections after each topic.

applications of math to art (although that is fascinating), but rather how to use art to introduce math concepts to children. You will need to find some posters, slides, or overheads to do this. Visit the Fine Arts Library, and explore and use your imagination. (If you cannot check out the slides from the library, let me know and I will borrow them for you.)

Additional Comments: Make sure the PSTs have some form of visual for the class for this topic. Typically, the PST's do well with this presentation. If there are struggles, though, challenge them to find concepts of line, point, ray, segment, polygon, infinity, dimension, or anything more you can think of in art that exists already. Again, this should not be an arty presentation ... there needs to be mathematical substance.

5. $*$ **A Conjecture on Area and Perimeter** (grade 5). See attached for description. First decide if the conjecture is true and then see your instructor (as a group) before settling on the rest of the project. Your presentation is about how you (as a teacher) would respond to the student in front of their class. What is your mathematical habit of the mind in responding? What do you believe she is thinking about? How would you explain the mathematics conceptually?

   Additional Comments: This is on the disc (for your reference). The PST's should be able to understand what the connection between area and perimeter is, but they will struggle to fill a presentation with the material. Having them construct different rectangles with unit squares and equal areas is a great conceptual way to see what happens to the perimeter when the area remains constant. You might also challenge the group to investigate what happens with other shapes (hexagons, octagons, etc). Be sure the PST's have something to say for this presentation.

6. **Volume of Cylinders and their Surface Area** (Grade levels 4-6 or later): "Case 4: Slippery Cylinders" in Katherine Merseth, editor, WINDOWS ON TEACHING MATH. Work through the case and then explain conceptually the various responses of the students, as well as the appropriate one. Also perform a *rice experiment* in your presentation.

   Additional Comments: Make sure they do the rice experiment to verify the difference in volume of the two figures. The PST's struggle with introducing this topic to the class, as well as the layout of the presentation. Furthermore, it is easy for them to get caught up in calculations, so be sure they avoid this. As a reminder to them, the formulas have already been covered in class, so they need not re-present that material. To challenge the PST's further, perhaps you could ask them to investigate how volume and surface area are related when you turn a sheet of paper into a torus.

7. **Algebraic Expressions** (Grade levels are non-college bound algebra II class). See "Case 1: Lost in Translation", in Katherine Merseth , editor, WINDOWS ON TEACHING MATH. This case study deals with trying to explain moving algebraic expressions from word descriptions to a class which has had math difficulties in the past. The algebra involves expressions like $2 - 5n = 12 + n$. Work through the case and then explain conceptually the various responses of the students as well as the appropriate one. The level of algebra is that of Chapter 8 in Long and DeTemple.

   Additional Comments: This topic is a hard one for the students to present on. Make sure they go through each of the cases. Challenge the PST's to put the article into their

own words. Also, have this group integrate the class into their presentation by asking them what they would write for the word problem. From experience, translating word problems into symbols can be difficult for a variety of students, so make sure this group realizes that.

8. **Probability and Data Analysis** (Early childhood): See the article " Data Analysis and Probability in the Early Childhood Curriculum", p. 147-166 of Jaunita Copley, THE YOUNG CHILD AND MATHEMATICS. Your presentation must include an instruction principle and assessment principle (see p. 166) and answer the question: "What would be the math content of a similar project in the next grades and how would you structure it?". What is it from MA 202 that is relevant to this article?

   Additional Comments: The article does not provide deep mathematical thought, but the PST's goal is to explain how this prepares a student for later grade levels (conceptually). The PST's can discuss the different activities discussed in this article, but should not consume time by demonstrating the activities in their presentation. This is a rare topic to be chosen.

9. **Statistics - Averages** (Grade 4): This project discusses how to compute averages by non-standard means (pun intended). "Fourth Graders Invent Ways of Computing Averages", Kamii, Pritchett, and Nelson in NCTM, PUTTING RESEARCH INTO PRACTICE, p. 232-237.

   Additional Comments: This topic is a great supplement to the concepts in statistics (also included on the disc). Each of the three methods should be discussed in the presentation. Encourage the PST's to invent their own ways of computing the mean, and to discuss with the class how each of these ways is conceptually equivalent to the actual formula for the mean. Again, this is a topic that is rarely chosen. Make sure the PST's do not bog their presentation down with calculations, but rather focus on what the different students did and why the math is correct.

10. **Algebraic Representations** (Grades 4 and 5) :Graphs and the stories they represent. Do activity 5 of Dolan, Williamson, Muri, and ACTIVITIES FOR ELEMENTARY MATHEMATICS TEACHERS, p. 148. Now formulate content oriented questions based on each of the five graphs which will change the shape of the graph given. Be explicit about what the resulting graph will look like. Which of these can be described by *algebraic formulas*?

    Additional Comments: There is not much in the way of content to present on, so the PST's will be challenged to come up with more. The PST's should have a good conceptual understanding of how graphs change according to the situation, and they should show the class some examples of what might happen if a situation changes. Challenge the PST's to break down a worded situation and describe what the resulting graph should look like. Perhaps the PST's could also come up with some situations on their own, and then try to graph the situation. Ask the question: "Why might figure A not be considered the height of a ball?". (answer: look at what time the ball starts to go up, and how do we account for the height of a person!)

11. **Figure in Space** (Grades 4-8): Present the material of Long and DeTemple, Section 11.3.

    Additional Comments: Keep in mind that this is a section from the book. There is alot of material to cover in this section, though, so you can still lecture this section if you feel like it. If you do, make sure the PST's do not regurgitate something you have already done. One suggestion for this topic is to have the PST's construct different polyhedra out of paper (there are many examples in this section, but challenge them to go further). Perhaps they could also construct non-polyhedra, and explain why the figures are not polyhedron. The PST's should not just list out different kinds of polyhedra . . . there should be some explanation as to how shapes can be different.

12. **Your Choice** (almost): Look at NLVM (see below for details), pick a manipulative that is relevant to MA 202, and explain its content and conceptual base to our class (audience: your fellow classmates). Please check with your instructor first to make sure that the topic and its mathematical depth are appropriate. (All grade levels).

REMEMBER:

1. Emphasis is on math content at the appropriate grade level throughout.
2. Look at the National Library for Virtual Manipulatives (NLVM) for Interactive Mathematics (http://matti.usu.edu/nlvm/nav/vlibrary.html) and use one in your presentation or use another manipulative. You are not required to use NLVM's manipulatives, though.
3. When there is student work in the article you read, you need to present it and explain conceptually why it is right or wrong.
4. All of the books and articles are on reserve in the Math Library, POT 065, under MA 202 and your instructor's name.

$*$:= there must be a presentation on each of these topics in each MA 202 section.

Expectations of the Instructor for the Presentation

(i) It is up to you as to how the groups, presentation topics, and peer mentors are assigned. But the groups and topics need to be assigned *at least* 3 weeks before the presentation, so that the groups have enough time to prepare for the presentations. Emphasis is also put on assigning the peer mentors *before* the presentations begin (not on the day of, as this will potentially mess up the presentation schedule).

(ii) You (the instructor) will be EXPECTED to meet with each group before their presentation. Be sure to read through the descriptions of the presentations before you meet with a group. After the group has constructed their presentation, you need to schedule a time to have them run their presentation by you. PST's in the past tended to overtalk, read verbatim from the book, re-teach something already taught by you or another group, teach strictly procedurally, not be prepared with demonstrations, etc. Meeting with the groups will help to avoid some of this. Also, some of the topics might seem "easy" to them, so make sure that they truly understand what is going on and are able to give a meaningful presentation.

(iii) Give the PST's a warning (when giving their presentations) if they are close to 7-10 minutes. This helps keep the class on schedule.

(iv) PST attendance during the presentation is REQUIRED. If a student does not show up, points need to be deducted from the presentation grade. If a student is being inconsiderate during the presentation, do not hesitate to ask them to leave. In your instructions to the PST's, make sure they realize that attendance is mandatory, and that points WILL be deducted if they are not present.

(v) Observe that 70% of the grade is your (instructor) assessment based upon the criterion given out. We suggest grading the group as a whole, as opposed to each individual presentation. From experience, though, there could be groups where it is evident that some members put more into it than others. In this situation, you may want to grade individually. Be sure to provide justification for the presentation grade.

(vi) To re-iterate, the peer mentor assessment should be turned in after all the presentations have been given. You will want to emphasize to the class that person A's assessment of person B will not affect person B's grade. Grading of person A's assessment will be purely on whether person A preformed an adequate assessment. Note that some students will not take this seriously (person B did *everything* right!), and person A's grade needs to suffer. In this case, the student did not do the assignment correctly. The assessments will not be communicated to the presenter, so encourage the students to be objective.

(vii) Since every topic will not be presented on, you should not test them over the presentation material. Test only over what you covered in class.

Expectations of the PST's for the Presentations

(i) It is the responsibility of the PST to get together with their group. If a PST is having trouble with a group, they need to tell the instructor, preferably before their presentation.

(ii) The PST needs to understand who the audience is for the presentation. This is not an education course, but rather a math content course. Thus their presentation should be heavily math based. Also, again, they are teaching to their fellow classmates, as opposed to an elementary class. Conceptual teaching should be emphasized above all else.

(iii) The presentations should be rehearsed. If there is a demonstration being performed within the presentation, the group needs to make sure that is ready. There should be no extended silent time.

(iv) Presentation time should not be consumed with writing on the board. PST's should have definitions/theorems ready beforehand, or have someone else write them up while they're speaking. "Simple ideas" that the class already knows, such as the definition of a triangle, should not be included in the presentation.

(v) The PST needs to (try to) go deeper into their subject than what is originally given. Many PST's like to keep to the bare minimum, and regurgitate what the book says. Challenge the PST's to conceptualize the ideas.

(vi) Objectivity should be used in assessing a fellow classmate (when doing the peer mentor assignment).

(vii) PST's need to show up for the presentation. While watching the presentation, the PST's need to show respect and civility towards their peers. Expect points to be deducted from the PST's grade if not present for the presentation.

(viii) Each PST needs to speak for 7 to 10 minutes! No exceptions!

NOTE: THIS PAGE SHOULD BE DUPLICATED AND GIVEN TO THE PST'S

5 Breakdown of Course

We take time in this last section to summarize this course week by week, as well as give a suggested point breakdown of the assignments. There will be 4 exams total (3 and a final), and you should shoot for 9-12 homework assignments. The PST's should be comfortable with not needing a calculator, so calculators are forbidden for all exams. The only other grade the PST's need to worry about is from the presentation. Observe that about 70% of the presentation is the actual presentation, and the other 30% is the peer assessment. Examples of old exams, syllabi, etc. are contained on a CD accompanying this packet. A *suggested* percentage breakdown is given below:

Exam 1:	15%
Exam 2:	15%
Exam 3:	15%
Final Exam:	30%
Presentation:	15%
Homework:	10%
Final Grade:	100%

Next, a week by week schedule is given. Note that the schedule may vary from instructor to instructor, but this gives a good idea of how the class should flow.

Week 1: Sections 8.1

Week 2: Sections 8.1 - 8.3 ; Homework #1

Week 3: Sections 8.3 - 9.1 ; Homework #2

Week 4: Sections 9.1 - 9.2 ; Homework #3; Withdraw deadline without a "W"

Week 5: Sections 9.2 - 9.3 ; **EXAM 1**

Week 6: Sections 10.1 - 10.2 ; Homework #4

Week 7: Sections 10.2 - 11.1 ; Homework #5

Week 8: Sections 11.1 - 11.2 ; Homework #6

Week 9: Sections 11.2 - 11.3 ; **EXAM 2** ; Withdraw deadline with a "W"

Week 10: Sections 12.1 - 12.2 ; Homework #7

Week 11: Sections 12.2 - 12.4 ; Homework #8

Week 12: Sections 13.1 - 13.2 ; Homework #9

Week 13: Sections 13.2 - 13.3 ; **EXAM 3**

Week 14: Presentations

Week 15: Presentations and review

Week 16: **FINAL EXAM**