Def: **coordinate axes** - two perpendicular number lines (one vertical [y-**axis**], one horizontal [x-**axis**]). Note that any point in the plane is uniquely determined by its distance above the x-axis (y-**coordinate**), and its distance to the right of the y-axis (x-**coordinate**). Each point is generally given as an **ordered pair** (a, b).

The axes divide the coordinate plane into four **quadrants**, and the point $(0, 0)$ where the axes intersect is called the **origin**.

Example 8.6 - Be able to plot points, and to determine the coordinates of plotted points.

Theorem: **The distance formula** - let P and Q be the points (x_1, y_1) and (x_2, y_2) respectively. The the distance between P and Q is $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Example 8.7 - Prove that the triangle with vertices $R(1, 4), S(5, 0)$ and $T(7, 6)$ is isosceles (ie. two of the sides are the same length).

Notation: $\overleftrightarrow{PQ}$ is the unique line through P and Q, $\overline{PQ}$ is the line segment with endpoints at P and Q, and PQ is the distance between P and Q (or the length of $\overline{PQ}$).

Slope is a measure of the steepness. It is generally thought of as "rise over run". When surveyors discuss the grade of a slope, they are just converting it to a percent - so a slope of 0.05 is a grade of 5%.

Def: Let $P(x_1, y_1)$ and $Q(x_2, y_2)$, with $x_1 \neq x_2$, be two points. Then the **slope** of $\overline{PQ}$ (or also $\overleftrightarrow{PQ}$) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Note: If $x_1 = x_2$ then the line between P and Q is vertical, and we say that the slope is *undefined*. Horizontal lines have slope equal to 0. You should be able to tell by looking if the slope of a given line is undefined, 0, positive or negative.

Example 8.8 - The points $P(-4, 3), Q(5, 6), R(5, -1)$ and $S(-1, -3)$ are the vertices of the polygon $PQRS$. Find the slope of each side of the polygon.

Example 8.9 - A conceptual basis for the equation of a line through $(2, 3)$ having slope $4/3$.

Def: **Point-Slope form** of the line through (x_1, y_1) with slope m is given $y - y_1 = m(x - x_1)$.

Def: **Slope-Intercept form** of the line with slope m and y-intercept b is given by $y = mx + b$.

Example 8.10 - Given a slope and y-intercept, be able to write an equation of the line and graph it.

Note: Horizontal lines have equations of the form $y = b$ and vertical lines have equations of the form $x = a$.

Example 8.11 - be able to find an equation for a line connecting two given points.

Def: **Two-Point form** of the line through (x_1, y_1) and (x_2, y_2), where $x_1 \neq x_2$ is given by $y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$.

Def: **General form** of the equation of a line is given by $Ax + By + C = 0$ where A, B, C are constants and A, B are not both 0.

Note: If $B \neq 0$ in the general form, you should be able to find the slope and y-intercept of the line in terms of A, B and C.

Def: Two lines l and m in the plane are **parallel** if they have no points in common, or if they are equal. We write $l \parallel m$ if l and m are parallel lines, and $l \nparallel m$ if they are not.

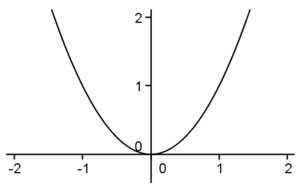
Note: There are only three alternatives for the intersection of two lines. They intersect every where (they are the same line), they intersect nowhere (they are distinct parallel lines), or they intersect in a single point (they are not parallel).

Theorem: Two lines in the plane are parallel if and only if they both have the same slope or both are vertical lines.
Proof: Case I - Vertical lines. Case II - By contradiction.

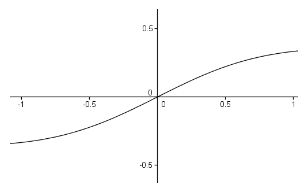
Example 8.12 - Algebra can be used to solve some geometry problems. In this instance, if we think of the triangle as being on the Cartesian plane, it becomes an easy problem.

Def: **Non-linear functions** are *not* able to be written in the form $f(x) = mx + b$ (where $m, b \in \mathbb{R}$).

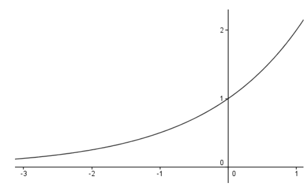
To graph non-linear functions by hand, you need to plot many points that are on the graph.



$g(x) = x^2$

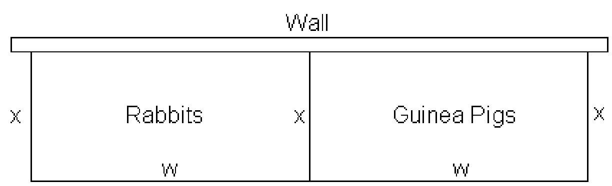


$h(x) = \frac{x}{x^2+2}$



$k(x) = 2^x$

Example 8.13 - The third grade class wants to make pens for its pet rabbits and guinea pigs. A parent has donated 24 feet of chain-link fencing material, which will be used to make two side-by-side identical rectangular pens along a wall. What dimensions of x and w will give the pens their largest total area?



Let x be the height of each pen, and w be the width of each. $x \geq 0, w \geq 0$. The area of the pen is $A = 2wx$, and to maximize our fencing we must have that $3x + 2w = 24$. This gives us $A = (24 - 3x)(x) = 24x - 3x^2$ (for $0 \leq x \leq 8$).

x	0	1	2	3	4	5	6	7	8
A	0	21	36	45	48	45	36	21	0

If we graph the function, we will see that the max value ($A = 48$) occurs at $x = 4$.
So we need $x = 4$ ft and $w = 6$ ft.

Section 8.2 HW: #5, 6a-d, 7, 9, 13c, 26, 33a, 40 (part of HW1)