

Def: **experimental probability** (P_e) - suppose an experiment is performed n times, and the outcome of the experiment is event E for r of those n times. Then the experimental probability $P_e(E)$ that E will occur on any given trial is $P_e(E) = \frac{r}{n}$.

◆ I flip a coin ten times, it lands on head 4 times. What is $P_e(H)$? $P_e(T)$? ($P_e(H) = 0.4, P_e(T) = 0.6$)

Theorem: **Law of Large Numbers** - As an experiment is performed repeatedly, the experimental probability of a particular outcome begins to approximate a fixed number.

Example: John Kerrich flipped a coin 10,000 times, recalculating the experimental probability after each flip. Although there were relatively long strings of H or T, overall it tended toward 0.5.

Example 10.1: Open the book to a random page and record the right hand page number. After 20 pages, what is the experimental probability that the page number is divisible by 3? ($7/20 = .35$)

Example 10.2: Determine P_e (a HS boy is between 68 and 70 inches tall). (.1875+.2375+.175=.60)

Example 10.3: Compute P_e (a student got in the 70s [a C] on the calculus final). ($6/28 = .21$)

Example 10.4: What is P_e (at most 1 head)? ($8/50 = .16$)

Def: **outcome** - a result of one trial of an experiment

Def: **sample space** S - the set of all outcomes of an experiment

Def: **event** E - a set of some of the outcomes of the experiment ($E \subset S$)

Def: **mutually exclusive events** - events A and B are mutually exclusive if the occurrence of one prevents the other from happening. ($A \cap B = \emptyset$)

When summing the roll of two dice, $S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$, we can say event A is rolling a 7, event B is rolling an 11, event E is rolling a 7 or an 11.

Example 10.5: Compute $P_e(7) = 12/50 = 0.24, P_e(11) = 2/50 = 0.04, P_e(7 \text{ or } 11) = 14/50 = 0.28$.
Show that $P_e(7) + P_e(11) = P_e(7 \text{ or } 11)$. ($0.24 + 0.04 = 0.28$)

Property: If A and B are mutually exclusive events, then $P_e(A \text{ or } B) = P_e(A) + P_e(B)$.

Example 10.6: When flipping a penny and a dime 50 times, A is the event 'the dime shows a head', B is the event 'the penny and the dime land same side up'. Compute $P_e(A), P_e(B), P_e(A \cup B)$.

This shows a more general property: $P_e(A \cup B) = P_e(A) + P_e(B) - P_e(A \cap B)$.

Def: **independent events** - events A and B are independent if the occurrence of one does not affect the occurrence (or non-occurrence) of the other.

Example 10.7: Compute $P_e(A) = 24/50 = .48, P_e(B) = 23/50 = .46, P_e(A \cap B) = 14/50 = .28$.
Show that $P_e(A \cap B) \approx P_e(A) \cdot P_e(B)$ ($0.28 \approx (0.48) \cdot (0.46) = 0.22$)

Property: If A and B are independent events, then $P_e(A \text{ and } B) \approx P_e(A) \cdot P_e(B)$.

Example 10.8: Estimating $P_e(A), P_e(B), P_e(C)$.

With a few basic assumptions, we can simulate other experimental probabilities - such as using three coins to determine the probability that a family with three children has at least one boy.

Section 10.1 HW: # 1, 6, 8, 19, 20, 28, 32 (part of HW3)