Notation: $n(A)$ is the number of elements in the set A.

Def: **theoretical probability** - the theoretical likelihood of an event happening. If all outcomes are equally likely, $P(E) = \frac{n(E)}{n(S)}$.

♦ Are there any examples why we would use experimental probability instead of theoretical probability?

♦ How many cards in an ordinary deck are diamonds or face cards? $(13 + 12 - 3 = 22)$

Theorem: **The Addition Principle of Counting** - if A and B are events, then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Example 10.10: In how many ways can you select a red card or an ace from an ordinary deck of playing cards? $(26 + 4 - 2 = 28)$

Example 10.11: Determine the number of ways of obtaining a score of 7 or 11 on a single roll of two dice. $(6 + 2 - 0 = 8)$

Example 10.12: A box of 40 chocolates contains 14 milk chocolates, 16 caramels, and 10 dark chocolates. In how many ways can you select a milk chocolate or a caramel from the box? $(14 + 16 = 30)$

Example 10.13: On an airplane from Frankfurt to Paris, all the people speak French or German. If 71 speak French, 85 speak German and 29 speak both French and German, how many people are on the plane? $(71 + 85 - 29 = 127)$

♦ Can we do this using a Venn diagram? (See slides.)

A bag of marbles contains three red numbered marbles (r_1, r_2, r_3) and one green marble (g). In Stage 1, draw a marble from the bag, keep it in your left hand. In Stage 2, draw a marble from the bag, keep it in your right hand.
♦ What is the sample space? (See slides.)
♦ In how many ways will the left and right hands both hold a red marble? (6)

Theorem: **The Multiplication Principle of Counting** - if A and B are outcomes of different stages of an experiment, the number of ways that A and B can occur together is $n(A \cap B) = n(A) \cdot n(B|A)$.

♦ In our last example, $n(B)$ can't be defined since B depends on Stage 1. If A and B are independent, $n(B|A) = n(B)$.

Example 10.14: How many ways can two aces be drawn in succession from a standard deck of 52 cards if:
(a) the first card is replaced randomly before the second card is drawn $(4 \cdot 4 = 16)$
(b) the first card is not replaced before the second card is drawn $(4 \cdot 3 = 12)$

Example 10.15: How many five letter code 'words' can be formed if:
(a) repetition of letters is allowed? $(26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 = 26^5 = 11,881,376)$
(b) repetition of letters is not allowed? $(26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 = 7,893,600)$

Def: **complementary event** $(\overline{E})$ - the event that E *does not* occur.

Theorem: Let $\overline{E}$ denote the complement of event $E \subseteq S$. Then $n(E) = n(S) - n(\overline{E})$.

Example 10.16: A fair coin is tossed 10 times. In how many ways can at least two heads appear?
$(n(S) = 2^{10} = 1024,\ n(\overline{E}) = 1 + 10 = 11,\ n(E) = 1024 - 11 = 1013)$

Section 10.2 HW: # 3, 4, 6, 9, 15, 18, 23, 28, 37 (part of HW3)