Def: **permutation** - an arrangement of a given number of objects from a specified set into an *ordered list*.
Def: **combination** - a selection of a given number of objects from a specified set into an *unordered list*.

Example 10.17: There are five members of the Math Club.

(*a*) In how many ways can we choose a president and treasurer? $\big(5 \cdot 4 = 20\big)$

(*b*) In how many ways can we choose two people for a committee? $\big(20/2 = 10\big)$

Notation: $P(n,r)$ is the number of permutations of r objects from a set of n, sometimes also nPr.
$C(n,r)$ is the number of combinations of r objects from a set of n, sometimes also nCr or $\binom{n}{r}$.

Def: **factorial** - multiplying a positive integer by every positive integer less than itself.
eg. $k! = k \cdot (k-1) \cdot (k-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1 \qquad 0! = 1$

Example 10.18: Compute each of the following:

- $1! = 1$
- $2! = 2$
- $3! = 6$
- $4! = 24$
- $4 \cdot 3! = 4! = 24$
- $(4 \cdot 3)! = 12!$
- $4!+3! = 24+6 = 30$
- $4!-3! = 24-6 = 18$
- $\frac{8!}{5!} = 8 \cdot 7 \cdot 6$
- $\frac{8!}{7!} = 8$
- $\frac{8!}{8!} = 1$
- $\frac{8!}{0!} = 8!$

Thm: $P(n,r) = \frac{n!}{(n-r)!} \qquad C(n,r) = \frac{P(n,r)}{r!} = \frac{n!}{r!(n-r)!}$

Formulas for Permutations and Combinations: 5-letter 'code words' with no repetition.

Example 10.19: Compute each of the following:

- $P(7,2) = \frac{7!}{5!} = 7 \cdot 6$
- $P(8,8) = \frac{8!}{0!} = 8!$
- $P(25,2) = \frac{25!}{23!} = 25 \cdot 24$
- $C(7,2) = \frac{7!}{2!5!} = \frac{7 \cdot 6}{2} = 21$
- $C(8,8) = \frac{8!}{0!8!} = \frac{1}{1} = 1$
- $C(25,2) = \frac{25!}{2!23!} = \frac{25 \cdot 24}{2} = 25 \cdot 12$

Example 10.20: The stamp club has nine members, including its president, Alicia. A four-person refreshment committee is to be formed.

- How many committees include Alicia? $\big(C(8,3) = \frac{8!}{3!5!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56\big)$
- How many committees do not include Alicia? $\big(C(8,4) = \frac{8!}{4!4!} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} = 7 \cdot 2 \cdot 5 = 70\big)$
- How many committees are there altogether? $\big(C(9,4) = \frac{9!}{4!5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1} = 3 \cdot 7 \cdot 6 = 126 = 70 + 56\big)$

Example 10.21: We must arrange 4 red flags, 3 blue flags, and 2 green flags. How many different ways can we do this (assuming that switching two same-colored flags creates the same arrangement).
$\big(C(9,4) \cdot C(5,3) \cdot C(2,2) = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4!} \cdot \frac{5 \cdot 4 \cdot 3}{3!} \cdot \frac{2 \cdot 1}{2!} = \frac{9!}{4!3!2!}\big)$

♦ How many different arrangements are there of the letters in ATLANTA? $\big(\frac{7!}{3!2!1!1!} = \frac{7 \cdot 6 \cdot 5 \cdot 4}{2} = 420\big)$

In class problems: p. 639 #26, p. 640 #30.

Homework 4 (due 3/2/10):
Section 10.3 # 2, 4, 5, 8, 10, 11, 13, 20, 21, 25a-c, 31ab, 32
Section 10.4 # 1, 3, 4, 5, 7, 9, 10, 35