Def: When all of the outcomes in the sample space S are equally likely, the **theoretical probability** (or just **probability**) of event E is $P(E) = \frac{n(E)}{n(S)}$

♦ The phrase *equally likely* is important. You must be careful as to what your sample space is. When rolling two dice, the sample space consists of 36 elements - not 11.

Example 10.23: Probability of getting a score of an 11 with two dice. $\left(2/36\right)$

Example 10.24: Probability of getting a score of 5 or 8 with two dice. $\left(9/36\right)$

Example 10.25: Probability of getting a face card or a diamond. $\left(22/52\right)$

Example 10.26: All 24 students in Mr. Henry's preschool are either 3 or 4 years old, as shown in the table at the top of the next page. A student is selected at random.

- What is the probability that the student is 3 years old? $\left(14/24\right)$
- What is the probability that the student is 3 years old, given that a boy was selected? $\left(8/11\right)$

Def: **conditional probability** - the probability that event B occurs given that event A has already occured: $P(B|A) = \frac{P(A\cap B)}{P(A)}$

This can be rewritten as: $P(A \cap B) = P(A) \cdot P(B|A)$.

If A and B are independent, then $P(B|A) = P(B)$.

Example 10.27: A red die and a white die are rolled. What is the probability of obtaining an even number on the red die and a multiple of three on the white die? $\left(3/6 \cdot 2/6 = 6/36\right)$

Example 10.28: An urn contains three identical red and two identical white balls. Two balls are drawn one after the other without replacement.

- What is the probability that the first ball is red? $\left(3/5\right)$
- What is the probability that the second ball is red, given that the first ball was red? $\left(2/4\right)$
- What is the probability that both balls are red? $\left(3/5 \cdot 2/4 = 6/20\right)$

Example 10.29: There are 10 boys and 13 girls in Mr. Fleck's fourth grade class, and 12 boys and 11 girls in Mrs. Patero's fourth grade class. A picnic committee of six people is selected at random from the total group of students in both classes.

- What is the probability that all committee members are girls? $\left(C(24,6)/C(46,6)\right)$
- What is the probability that all committee members are girls, given that all come from Mr. Fleck's class? $\left(C(13,6)/C(23,6)\right)$
- What is the probability that the committee has 3 girls and 3 boys? $\left(C(22,3) \cdot C(24,3)/C(46,6)\right)$
- What is the probability that the committee has 3 girls and 3 boys given that Mary Akers and Ann-Marie Harboth are on the committee? $\left(C(22,3) \cdot C(22,1)/C(44,4)\right)$

Thm: Let A and $\overline{A}$ be complementary events (ie $A \cup \overline{A} = S$ and $A \cap \overline{A} = \emptyset$), then $P(A) = 1 - P(\overline{A})$.

Example 10.30: Compute the probability of obtaining a score of at least 4 on a single roll of two dice. $\left(1 - 3/36 = 33/36\right)$

Example 10.31: A hand of 5 cards is drawn from a standard deck. What is the probability that both colors (red and black) are represented in the hand?

$\left(P(\overline{E}) = P(R \cup B) = P(R) + P(B) = C(26,5)/C(52,5) + C(26,5)/C(52,5) \approx 0.05 \text{ then } P(E) = 0.95\right)$

Theorem: Properties of Probability

Def: **odds** in favor of event E - the ratio of the number of ways E happen to the number of ways E doesn't happen $= n(A)$ to $n(\overline{A})$ or $n(A) : n(\overline{A})$.

Example 10.33: Determining the odds of rolling a 7 or 11 $\big($8 to 28$\big)$

Example 10.34: If the odds in favor of event E are 5 to 4, compute $P(E) = 5/9$ and $P(\overline{E}) = 4/9$.

Example 10.35: Given the $P(E) = 22/52$ compute the odds in favor of E $(22 : 30)$, and the odds against E $(30 : 22)$.

Def: **expected value** of an experiment - the 'typical' amount you expect to obtain from it. If the possible values of the experiment are $v_1, v_2, \dots, v_n$, and they occur with probabilities $p_1, p_2, \dots, p_n$ respectively, then $e = p_1v_1 + p_2v_2 + \dots + p_nv_n$.

Expected value of a dice roll: $1/6 \cdot 1 + 1/6 \cdot 2 + 1/6 \cdot 3 + 1/6 \cdot 4 + 1/6 \cdot 5 + 1/6 \cdot 6 = 3.5$

Example 10.36: An American roulette wheel has 38 numbered compartments, green 0 and 00, and numbers 1 through 36 (alternating red and black). How much do you expect to win if you consistently bid \$5 on red? $\big(\frac{18}{38} \cdot 5 + \frac{20}{38} \cdot (-5) = -0.26\big)$

Example 10.37: Suppose we have a game where you pay \$21 flip three coins at once. You win \$100 if all three land heads, you win \$20 if two land hands, and you win nothing if more than one lands tails. Would you play the game? $\big(\ 1/8 \cdot 100 + 3/8 \cdot 20 + 4/8 \cdot 0 = 20\ \big)$

Homework 4: (due 3/2/10)
Section 10.3 # 2, 4, 5, 8, 10, 11, 13, 20, 21, 25a-c, 31ab, 32
Section 10.4 # 1, 3, 4, 5, 7, 9, 10, 35