Def: **transformation of the plane** - a one-to-one correspondence of the plane to itself
Def: **image** of a point P - where P goes under the transformation
Def: **preimage** of a point P' - where P' came from under the transformation
Def: **rigid motion (or isometry)** - a transformation which preserves distance
Def: **equivalent transformations** - have the same net outcome

Four basic rigid motions: translations, rotations, reflections, glide-reflections

Def: **translation (or slide)** - all points in the plane are moved the same distance in the same direction
Def: **slide arrow** - shows how to move the points

Example 13.1 - Finding the image under a translation

Def: **rotation (or turn)** - one point (the **turn center (or center of rotation)**) is held fixed, the rest of the points in the plane are rotated the same number of degrees (the **turn angle (or angle of rotation)**) around that point.
Def: **turn arrow** - shows how much and where to rotate

Example 13.2 - Finding images under rotations

Def: **reflection (or flip or mirror reflection)** - mirroring every point perpendicularly over a line (the **line of reflection (or mirror line)**)

Example 13.3 - Finding images under reflections

Def: **glide-reflection** - a glide followed by a reflection

Example 13.4 - Determining a Glide-Reflection

♦ Note: The rotations $-120°$ and $240°$ are equivalent. Two consecutive $180°$ rotations make an **identity transformation**. Two successive flips over the same line of reflection is also an identity transformation.

Example 13.5: Consider two parallel lines of reflection. What is the net outcome of reflecting over each?
(If d is the distance between them, the net outcome is a translation by $2d$ perpendicular to the lines.)

♦ Note: If the two lines of reflection intersect (at an angle of $x°$), the net outcome is a rotation about their intersection point by $2x°$.

Theorem (Net Outcome of Reflections in Distinct lines):
- The net outcome of reflections across two parallel lines is equivalent to a translation perpendicular to the lines and twice the directed distance from the first line to the second.
- The net outcome of reflections across two intersecting lines is equivalent to a rotation about their point of intersection through an angle twice the directed angle from the first line to the second.

Theorem: The net outcome of reflections across three distinct lines is equivalent to either:
- a reflection (if all three are parallel or concurrent)
- a glide-reflection (if all three are neither parallel nor concurrent)

Theorem: Any rigid motion of the plane is equivalent to one of the four basic rigid motions: a translation, a rotation, a reflection, or a glide-reflection.

♦ To identify the type of rigid motion, first decide whether direction has been preserved or reversed. Translations and Rotations preserve direction, Reflections and Glide-Reflections reverse it.

Example 13.6: Classifying Rigid Motions

Def: Let O be a point in the plane, and k a positive real number. A **dilation (or size transformation)** with **center** O and **scale factor** k is the transformation that takes each point $P \neq O$ of the plane to the point P' on the ray $\overrightarrow{OP}$ for which $OP' = k \cdot OP$ and takes the point O to itself.
Def: If k is larger than 1 the dilation is an **expansion**.
Def: If k is smaller than 1 the dilation is a **contraction**.

Theorem: Under a dilation with scale factor k, the distance between any two image points is k times the distance between their preimage points.

Def: A transformation is a **similarity transformation** if and only if it is a sequence of dilations and rigid motions.
Def: Two figures F and G are **similar** if and only if there is a similarity transformation that takes one figure onto the other figure.

Homework 8 (due 4/13/10):

- Section 13.1 # 2b, 3b, 4, 5a, 9, 10, 11, 15
- Section 13.2 # 6, 9, 13, 43