1. **Figures in Space:** This project focuses on three-dimensional geometry, a topic which has recently been added to the K-5 curriculum. Using Section 11.3 of [3], and other resources you may find, your presentation should clearly introduce the following:

    - The definition of a polyhedron and Euler's Formula for Polyhedra. Be sure to include examples of polyhedra, an investigation of Euler's Formula for Polyhedra, and Example 11.18 in Section 11.3 of [3] (or something similar).
    - You may want to include examples of polyhedra in nature, architecture, etc., the idea of planar networks (found in Section 11.4 in [3]) and a proof of Euler's Formula for Polyhedra, nets of polyhedra, the construction of polyhedra, or the Platonic solids.

2. **The Pythagorean Theorem:** This project is designed to go beyond what we will have covered in Section 12.3 of [3] on the Pythagorean Theorem.

    - Using resources on the internet or other sources you may find, you should present at least 2 alternate proofs of the Pythagorean Theorem.
    - You may want to include examples of the Pythagorean Theorem throughout history, investigate how this formula behaves when a "non-right" triangle is used, or see if there are similar results for other polygons.

3. **Statistics - Averages:** This project discusses how to compute averages by non-standard means. Explore the idea of weighted and geometric averages through the article "Which Mean Do You Mean?" [4].

    - Your presentation should provide motivation for using these different means and examples to clearly illustrate how they are computed.
    - You may want to include examples beyond the ones in the article where these new means are applicable, create or find other ways to compute a central or "average" value, or extend these ideas to find other ways to measure variability.

4. **Volumes of Cylinders and their Surface Areas:** In this presentation, you will explore the concepts of volume and surface area, and investigate how they are related. Read "Case 4: Slippery Cylinders" in [5]. Your presentation should include

- a demonstration of the activity. Don't focus on calculations; instead, show that the volumes differ by filling each cylinder with an equal amount of rice. Also explain why their surface areas are equal. Discuss how the volume formula suggests which cylinder has the bigger volume, and state the relationship between surface area and volume.
- You may consider some of the following questions to help deepen your understanding: Using Ryan's question (p. 32) as a guide, what other types of geometry concepts are related to this one?
- How could a similar activity be designed using other geometric figures?
- What are the advantages and disadvantages of calculating the volumes vs. doing a rice experiment?
- Can you give an example where a cylinder with smaller surface area than another still has greater volume?

5. **Networks:** This project focuses on network theory (also commonly referred to as graph theory). Using Section 11.4 of [3], and other resources you may find, your presentation should clearly introduce the following:

   - The definition of a networks, including definitions related to networks (such as the definitions contained in Section 11.4 of [3]).
   - Euler's Traversability theorem and an outline of why it is true, including relevant examples.
   - Explain the applicability to the K onigsberg Bridge Problem and perhaps how to construct a network from the map of K onigsberg.
   - The definition of a planar network (including examples) and Euler's Formula for Connected Planar Networks.

6. **Polygon Properties and Tiling:** A tiling is loosely a way to cover or fill one geometric object with other smaller geometric objects. Section 13.3 of [3] discusses tilings in their various guises (polygonal, regular, etc.), their applications and their appearance in art and nature. Tilings appear explicitly as part of the elementary mathematics curriculum since they can be used to prove facts about polygons, illustrate symmetries (similarity transformations) in

the plane and appear in nature. A reference for tiling at the elementary level is "Connected Mathematics: Shapes and Designs" [2].

After your presentation, you and your fellow students should be able to answer questions like the following.

- How can one construct tilings using symmetries? That is, how can one start out with a polygon or other admissible figure and copy and move it to get a tiling?
- If we only consider regular polygons–which tile the the plane? Why is this?
- Using irregular tiling, can you prove that the interior angles of a triangle sum to 180°
- Is there alternate proof of the formula for interior angles of a regular polygon using tilings? It might be helpful to compare this to other ways of proving interior angle formulae.
- What else are tilings good for mathematically.

7. **Symmetries:** Symmetry in important mathematically and in nature. Symmetry as a rigid motion gives another way of classifying objects as geometrically the same. The basic definitions of symmetry should already be familiar from the lectures (that is Section 13.2 of [3]). As with the previous topic, the notion of symmetry appears explicitly in the elementary mathematics curriculum. A reference for symmetries at the elementary level is "Connected Mathematics: Kaleidscopes, Hubcaps, and Mirrors" [1].

   A presentation on this topic should explore the following.

   - What are the different symmetries present in the regular polygons? How does this relate to their various angle measurements? How do the symmetries relate? For example, how do the symmetries of a square and a rectangle relate?
   - Can we have non-polygonal plane figures with symmetry? What kind of symmetry does a circle have?
   - Given a set of symmetries in the plane, can one draw a figure which has all those symmetries?
   - Can one use algebra to to express symmetries of figures in the plane?

- Give real world examples of where symmetry is crucial.

8. **Understanding Polygons:** Since tracing polygons freehand fails to force one to obey the constraint that polygons must have sides consisting of only line segments we need other ways to present in polygons in a mathematically accurate way. Constructing tactile polygons is a useful way to develop intuition about the underlying mathematics. That is how do angles, side length and shape of a polygon relate. One should use "Connected Mathematics: Shapes and Designs" from [2] as a guide and starting point.

   To present this topic one should be able to answer the following.

   - In describing the measurement of angles our text talks about motion. How might this be illustrated with polystrips or other manipulatives in a mathematically correct way?
   - Given different polystrip constructions of plane figures, how can they move? Are there some polystrip constructions that can't move?
   - Moving the vertices of a polystrip construction without disconnecting the figure is not a rigid motion (see Section 13.1 of [3]). Does this give another way to distinguish polygons in the plane? Is there an algebraic representation of these transformations?
   - How does changing a side of a polystrip change the whole polystrip.
   - What could you do in higher dimensions to illustrate polyhedra?
   - Does motion of polystrips appear anywhere in the real world?

References

[1] Glenda Lappan et al. *Connected Mathematics: Kaleidoscopes, Hubcaps, and Mirrors–Symmetry and Transformations*. Pearson, 2004.

[2] Glenda Lappan et al. *Connected Mathematics: Shapes and Designs–Two-Dimension Geometry*. Pearson, 2004.

[3] Calvin Long, Duane DeTemple, and Richard Millman. *Mathematical Reasoning for Elementary Teachers*. Pearson, 5th edition, 2009.

[4] André Lubecke. Which mean do you mean? *Mathematics Teacher*, January 1991.

[5] Katherine Merseth. *Windows on Teaching Math: Case Studies in Middle and Secondary Classrooms*. Teachers College Press, 2003.