ASSIGNMENT 4

05-February-2007

1. Let f be integrable on $[a, b]$, and suppose that g is a function on $[a, b]$ so that $f(x) = g(x)$ except for finitely many $x \in [a, b]$. Show that g is integrable and that $\int_a^b f = \int_a^b g$.
   HINT: You will not want to use the properties of the integral, only the definitions and theorems through Theorem 4.4.

2. Let f be integrable on $[a, b]$ and let $c \in [a, b]$. Prove that $\int_c^c f = 0$.

3. Calculate $\lim_{x \to 0} \frac{1}{x} \int_0^x e^{t^2}\, dt$. HINT: Think l'Hospital.

4. Let f be defined as follows:
$$f(t) = \begin{cases} t & \text{for } t < 0, \\ t^2 + 1 & \text{for } 0 \le t \le 2, \\ 4 & \text{for } t > 2. \end{cases}$$
   (a) Determine the function $F(x) = \int_0^x f(t)\, dt$.
   (b) Sketch F. Where is F continuous?
   (c) Where is F differentiable? Find F' at all points of differentiability.

5. Let f be a continuous function on $\mathbb{R}$ and define
$$F(x) = \int_{x-1}^{x+1} f(t)\, dt \quad \text{for } x \in \mathbb{R}.$$
   Show that F is differentiable on $\mathbb{R}$ and compute F'.

6. Use the last example in the notes to show that
$$\int_0^{1/2} \arcsin x\, dx = \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1.$$

7. Let g be a strictly increasing continuous function mapping $[0, 1]$ to $[0, 1]$. Give a geometric argument showing
$$\int_0^1 g(x)\, dx + \int_0^1 g^{-1}(u)\, du = 1.$$

No WebWork assignment this week.