Quiz 2 — 09/15/16

Answer all questions in a clear and concise manner. Answers that are without explanations or are poorly presented may not receive full credit.

1. Compute the improper integral $\displaystyle\int_1^\infty \frac{1}{(2x+1)^3}dx$.

Solution:

$$\int_1^\infty \frac{1}{(2x+1)^3} = \lim_{B\to\infty}\int_1^B \frac{1}{(2x+1)^3}$$

Now let $u = 2x+1$ so $du = 2dx$.

$$\begin{aligned}
\lim_{B\to\infty}\int_1^B \frac{1}{(2x+1)^3} &= \lim_{B\to\infty}(1/2)\int_3^{2B+1}\frac{1}{u^3} \\
&= \lim_{B\to\infty}(-1/4)(1/u^2)|_1^{2B+1} \\
&= \lim_{B\to\infty}(-1/4)\left(\frac{1}{(2B+1)^2}-\frac{1}{3^2}\right) \\
&= 1/36
\end{aligned}$$

2. Let $f(x) = x^3$. What is the smallest number of sub-intervals n we must use on the interval $[0,2]$ so that the error in the trapezoidal approximation E_T of $\int_0^2 f(x)dx$ is less than .01? Hint: $|E_T| \le \frac{K(b-a)^3}{12n^2}$, where a and b are the endpoints of the interval and $|f''(x)| \le K$ for all $a \le x \le b$.

Solution: $f''(x) = 6x$, so $|f''(x)| \le 12$ on the interval $[0,2]$ since $6x$ is increasing. Now we plug in to the error forumula:

$$\begin{aligned}
\frac{6(2)^3}{12n^2} &\le .01 \\
\frac{4}{n^2} &\le .01 \\
n^2 &\ge 400 \\
n &\ge 20
\end{aligned}$$