Quiz 4 Solutions— 10/6/16

Answer all questions in a clear and concise manner. Answers that are without explanations or are poorly presented may not receive full credit.

1. Determine whether the series $\sum_{n=2}^{\infty} \frac{cos(n\pi)}{\sqrt{n}}$ is convergent or divergent.

Note that $cos(n\pi) = (-1)^n$.
Then $\sum_{n=2}^{\infty} \frac{cos(n\pi)}{\sqrt{n}}$ is an alternating series.
First, $a_n = 1/\sqrt{(n)}$ and $lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$.
Need to show that $a_{n+1} < a_n$
$\frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}}$
$\sqrt{n} < \sqrt{n+1}$, and thus a_n is decreasing.
Therefore, we have convergence by the Alternating Series Test.

2. Determine whether the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2}}$ converges absolutely, converges conditionally or diverges.

Again we have an alternating series, and $a_n = \frac{1}{n^{2/3}}$. And the $lim_{n\to\infty} \frac{1}{n^{2/3}} = 0$
Also need to show that $a_{n+1} < a_n$
$\frac{1}{(n+1)^{2/3}} < \frac{1}{n^{2/3}}$
$n^{2/3} < (n+1)^{2/3}$
$n < n+1$
Therefore our series is convergent, Now consider $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^2}}$. This is a p-integral with $p < 1$ and so this series diverges.
Thus we have conditional convergence.