

Quiz 5 — 10/13/16

Answer all questions in a clear and concise manner. Answers that are without explanations or are poorly presented may not receive full credit.

1. Determine the radius of convergence of the following series:

$$\sum_{n=1}^{\infty} \frac{(2x)^n}{n^2}$$

Applying the ratio test, let $a_n = \frac{(2x)^n}{n^2}$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(2x)^{n+1}}{(n+1)^2} * \frac{n^2}{(2x)^n} \right| = 2|x|$$

For the series to be convergent, we need $2|x| < 1$

$$2|x| < 1 \Rightarrow -1 < 2x < 1 \Rightarrow -\frac{1}{2} < x < \frac{1}{2}$$

The radius of convergence is $\frac{1}{2}$

2. Find a power series for the function below (center at 0)

$$f(x) = \frac{4}{x+2}$$

Recall the formula that $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

$$\text{Rewrite } \frac{4}{2+x} = \frac{4}{2-(-x)} = \frac{2}{1-(-x/2)}$$

$$\frac{2}{1-(-x/2)} = \sum_{n=0}^{\infty} 2\left(\frac{-x}{2}\right)^n$$

$$\text{Finally, we have } f(x) = \sum_{n=0}^{\infty} 2\left(\frac{-x}{2}\right)^n$$