MA 114 Worksheet #04: Special Trig Integrals

1. Compute the following integrals:

   (a) $\displaystyle\int_0^2 \frac{u^3}{\sqrt{16-u^2}}\,du$

   (b) $\displaystyle\int \frac{1}{x^2\sqrt{25-x^2}}\,dx$

   (c) $\displaystyle\int_0^{\pi/2} \cos^2(x)\,dx$

   (d) $\displaystyle\int \sqrt{\cos x}\,\sin^3 x\,dx$

   (e) $\displaystyle\int_0^{2\pi} \sin^2\left(\tfrac{1}{3}\theta\right)\,d\theta$

   (f) $\displaystyle\int_0^{\pi/2} (2-\sin\theta)^2\,d\theta$

   (g) $\displaystyle\int \frac{\sqrt{1-x^2}}{x^4}\,dx$

   (h) $\displaystyle\int_0^3 \frac{x}{\sqrt{36-x^2}}\,dx$. Hint: Use the substitution $x = 6u$.

   (i) $\displaystyle\int_0^{1/2} x\sqrt{1-4x^2}\,dx$. Hint: Substitute $x = u/2$.

2. Let $r > 0$. Consider the identity

$$\int_0^s \sqrt{r^2-x^2}\,dx = \frac{1}{2}r^2\arcsin(s/r) + \frac{1}{2}s\sqrt{r^2-s^2}$$

   where $0 \le s \le r$.

   (a) Plot the curves $y = \sqrt{r^2-x^2}$, $x = s$, and $y = \dfrac{x}{s}\sqrt{r^2-x^2}$.

   (b) Using part (a), verify the identity geometrically.

   (c) Verify the identity using trigonometric substitution.