MA 114 Worksheet #09: Recursive sequences & Series

1. Write out the first five terms of Let

   (a) $a_0 = 0$, $a_1 = 1$ and $a_{n+1} = 3a_{n-1} + a_n{}^2$.

   (b) $a_1 = 6$, $a_{n+1} = \dfrac{a_n}{n}$.

   (c) $a_1 = 2$, $a_{n+1} = \dfrac{a_n}{a_n + 1}$.

   (d) $a_1 = 1$, $a_{n+1} = \sqrt{\left(\dfrac{2}{a_n}\right)^2 + 1}$.

   (e) $a_1 = 2$, $a_2 = 1$, and $a_{n+1} = a_n - a_{n-1}$.

2. (a) For what values of x does the sequence $\{x^n\}_{n=1}^{\infty}$ converge?

   (b) For what values of x does the sequence $\{n^x\}_{n=1}^{\infty}$ converge?

   (c) If $\lim\limits_{n\to\infty} b_n = \sqrt{2}$, find $\lim\limits_{n\to\infty} b_{n-3}$.

3. (a) Determine whether the sequence defined as follows is convergent or divergent:

   $$a_1 = 1 \qquad a_{n+1} = 4 - a_n \qquad \text{for } n > 1.$$

   (b) What happens if the first term is $a_1 = 2$?

4. A fish farmer has 5000 catfish in his pond. The number of catfish increases by 8% per month and the farmer harvests 300 catfish per month.

   (a) Show that the catfish population P_n after n months is given recursively by

   $$P_n = 1.08P_{n-1} - 300 \qquad P_0 = 5000.$$

   (b) How many catfish are in the pond after six months?