MA 114 Worksheet #14: Power Series

1. (a) Give the definition of the radius of convergence of a power series $\sum_{n=0}^{\infty} a_n x^n$
   (b) For what values of x does the series $\sum_{n=1}^{\infty} 2\big(\cos(x)\big)^{n-1}$ converge?
   (c) Find a formula for the coefficients c_k of the power series $\frac{1}{0!}+\frac{2}{1!}x+\frac{3}{2!}x^2+\frac{4}{3!}x^3+\cdots$.
   (d) Find a formula for the coefficients c_n of the power series $1+2x+x^2+2x^3+x^4+2x^5+x^6+\cdots$.
   (e) Suppose $\lim_{n\to\infty} \sqrt[n]{|c_n|} = c$ where $c \neq 0$. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$.
   (f) Consider the function $f(x) = \frac{5}{1-x}$. Find a power series that is equal to $f(x)$ for every x satisfying $|x| < 1$.
   (g) Define the terms *power series*, *radius of convergence*, and *interval of convergence*.

2. Find the radius and interval of convergence for
   (a) $\sum_{n=0}^{\infty} \frac{(-1)^n n}{4^n}(x-3)^n$.
   (b) $4\sum_{n=0}^{\infty} \frac{2^n}{n}(4x-8)^n$.
   (c) $\sum_{n=0}^{\infty} \frac{x^{2n}}{(-3)^n}$.
   (d) $\sum_{n=0}^{\infty} n!(x-2)^n$.
   (e) $\sum_{n=0}^{\infty} (5x)^n$
   (f) $\sum_{n=0}^{\infty} \sqrt{n}x^n$
   (g) $\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n}}$
   (h) $\sum_{n=0}^{\infty} \frac{x^n}{3^n \ln n}$
   (i) $\sum_{n=0}^{\infty} \frac{(x=2)^n}{n^n}$
   (j) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n^4}$
   (k) $\sum_{n=0}^{\infty} \frac{(5x)^n}{n^3}$

3. Use term by term integration and the fact that $\int \frac{1}{1+x^2}\,dx = \arctan(x)$ to derive a power series centered at $x=0$ for the arctangent function. HINT: $\frac{1}{1+x^2} = \frac{1}{1-(-x^2)}$.

4. Use the same idea as above to give a series expression for $\ln(1+x)$, given that $\frac{dx}{1+x} = \ln(1+x)$. You will again want to manipulate the fraction $\frac{1}{1+x} = \frac{1}{1-(-x)}$ as above.

5. Write $(1+x^2)^{-2}$ as a power series. HINT: use term by term differentiation.