

MA 114 Worksheet #15: Taylor & Maclaurin Series

- Suppose that $f(x)$ has a power series representation for $|x| < R$. What is the general formula for the Maclaurin series for f ?
 - Suppose that $f(x)$ has a power series representation for $|x - a| < R$. What is the general formula for the Taylor series for f about a ?
 - Let $f(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4$. Find the Maclaurin series for f .
 - Let $f(x) = 1 + 2x + 3x^2 + 4x^3$. Find the Taylor series for $f(x)$ centered at $x = 1$.
- Assume that each of the following functions has a power series expansion. Find the Maclaurin series for each. Be sure to provide the domain on which the expansion is valid.
 - $f(x) = \ln(1 + x)$
 - $f(x) = xe^{2x}$
- Use a known Maclaurin series to obtain the Maclaurin series for the given function. Specify the radius of convergence for the series.
 - $f(x) = \frac{x^2}{1 - 3x}$
 - $f(x) = e^x + e^{-x}$
 - $f(x) = e^{-x^2}$
 - $f(x) = x^5 \sin(3x^2)$
 - $f(x) = \sin^2 x$.
HINT: $\sin^2 x = \frac{1}{2}(1 - \cos(2x))$
- Find the following Taylor expansions about $x = a$ for each of the following functions and their associated radii of convergence.
 - $f(x) = e^{5x}$, $a = 0$.
 - $f(x) = \sin(\pi x)$, $a = 1$.
- Differentiate the series in 1(b) to find a Taylor series for $\cos(x)$.
- Use Maclaurin series to find the following limit: $\lim_{x \rightarrow 0} \frac{x - \tan^{-1}(x)}{x^3}$.
- Approximate the following integral using a 6th order Taylor polynomial for $\cos(x)$:
$$\int_0^1 x \cos(x^3) dx$$
- Use power series multiplication to find the first three terms of the Maclaurin series for $f(x) = e^x \ln(1 - x)$.