

## MA 114 Worksheet #17: Average value of a function

1. Write down the equation for the average value of an integrable function  $f(x)$  on  $[a, b]$ .
2. Find the average value of the following functions over the given interval.

(a)  $f(x) = x^3, [0, 4]$

(b)  $f(x) = x^3, [-1, 1]$

(c)  $f(x) = \cos(x), \left[0, \frac{\pi}{6}\right]$

(d)  $f(x) = \frac{1}{x^2 + 1}, [-1, 1]$

(e)  $f(x) = \frac{\sin \pi/x}{x^2}, [1, 2]$

(f)  $f(x) = e^{-nx}, [-1, 1]$

(g)  $f(x) = 2x^3 - 6x^2, [-1, 3]$

(h)  $f(x) = x^n$  for  $n \geq 0, [0, 1]$

3. In a certain city the temperature (in  $^{\circ}F$ )  $t$  hours after 9 am was modeled by the function  $T(t) = 50 + 14 \sin \frac{\pi t}{12}$ . Find the average temperature during the period from 9 am to 9 pm.
4. The velocity  $v$  of blood that flows in a blood vessel with radius  $R$  and length  $l$  at a distance  $r$  from the central axis is

$$v(r) = \frac{P}{4\eta l}(R^2 - r^2)$$

where  $P$  is the pressure difference between the ends of the vessel and  $\eta$  is the viscosity of the blood. Find the average velocity (with respect to  $r$ ) over the interval  $0 < r < R$ . Compare the average velocity with the maximum velocity.

5. Breathing is cyclic and a full respiratory cycle from the beginning of inhalation to the end of exhalation takes about 5 s. The maximum rate of air flow into the lungs is about 0.5 L/s. This explains, in part, why the function  $f(t) = \frac{1}{2} \sin(2\pi t/5)$  has often been used to model the rate of air flow into the lungs. Use this model to find the volume of inhaled air in the lungs at time  $t$ . Then use this new function to compute the average volume of inhaled air in the lungs in one respiratory cycle.