

MA 114 Worksheet #22: Parametric Curves

1. (a) How is a curve different from a parametrization of the curve?
- (b) Suppose a curve is parameterized by $(x(t), y(t))$ and that there is a time t_0 with $x'(t_0) = 0$, $x''(t_0) > 0$, and $y'(t_0) > 0$. What can you say about the curve near $(x(t_0), y(t_0))$?
- (c) What parametric equations represent the circle of radius 5 with center $(2, 4)$?
- (d) Represent the ellipse $\frac{x^2}{a^2} + \frac{y^2}{c^2} = 1$ with parametric equations.
- (e) Do the two sets of parametric equations

$$y_1(t) = 5 \sin(t), \quad x_1(t) = 5 \cos(t), \quad 0 \leq t \leq 2\pi$$

and

$$y_2(t) = 5 \sin(t), \quad x_2(t) = 5 \cos(t), \quad 0 \leq t \leq 20\pi$$

represent the same parametric curve? Discuss.

2. Consider the curve parametrized by $c(t) = (\sin(t) + \frac{t}{\pi}, (\frac{t}{\pi})^2)$, for $0 \leq t \leq 2\pi$.
 - (a) Plot the points given by $t = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{3\pi}{2}, 2\pi$.
 - (b) Consider the derivatives of $x(t)$ and $y(t)$ when $t = \frac{\pi}{2}$ and $t = \frac{3\pi}{2}$. What does this tell you about the curve near these points?
 - (c) Use the above information to plot the curve.
3. Find a Cartesian equation for the following parametric curves. Sketch the curves to see if you solved them correctly.
 - (a) $x = \sqrt{t}, y = 1 - t$.
 - (b) $x = 3t - 5, y = 2t + 1$.
 - (c) $x = \cos(t), y = \sin(t)$.
4. Represent each of the following curves as parametric equations traced just once on the indicated interval.
 - (a) $y = x^3$ from $x = 0$ to $x = 2$.
 - (b) $\frac{x^2}{4} + \frac{y^2}{9} = 1$.
5. A particle travels from the point $(2, 3)$ to $(-1, -1)$ along a straight line over the course of 5 seconds. Write down a set of parametric equations which describe the position of the particle for any time between 0 and 5 seconds.