MA 114 Worksheet #27: Differential equations & Direction fields

1. (a) Is $y = \sin(3x) + 2e^{4x}$ a solution to the differential equation $y'' + 9y = 50e^{4x}$? Explain why or why not.

   (b) Explain why every solution of $dy/dx = y^2 + 6$ must be an increasing function.

   (c) What does is mean to say that a differential equation is linear or nonlinear.

2. Find all values of α so that $y(x) = e^{\alpha x}$ is a solution of the differential equation $y'' + y' - 12y = 0$.

3. Match the differential equation with its slope field. Give reasons for your answer.

$$y' = 2 - y \qquad y' = x(2 - y) \qquad y' = x + y - 1 \qquad y' = \sin(x)\sin(y)$$

(a) Slope field I

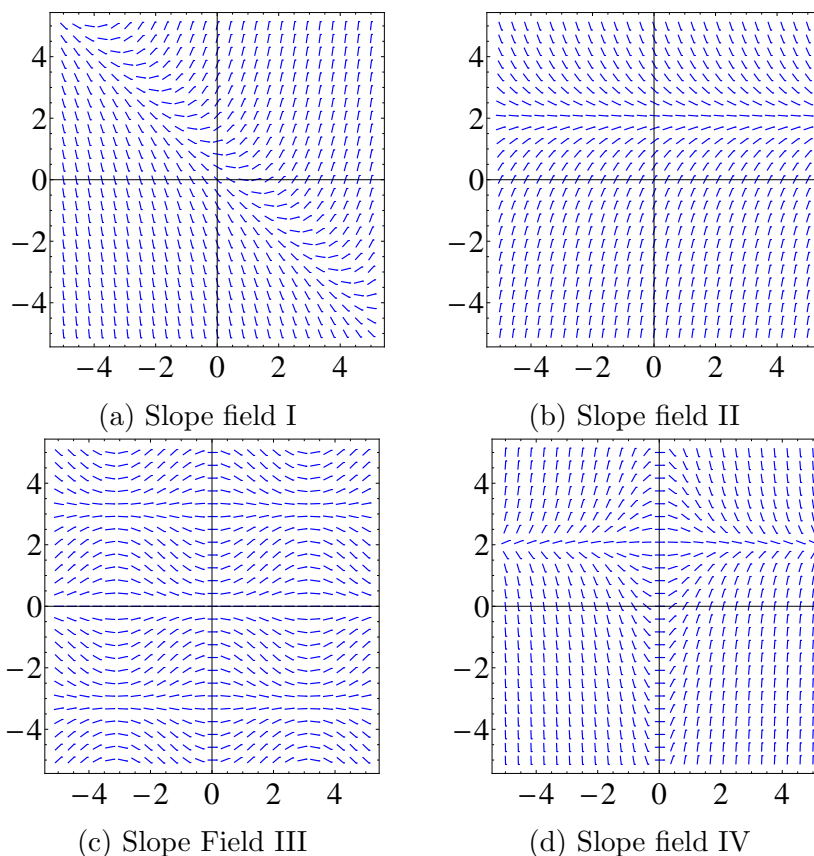(b) Slope field II

(c) Slope Field III

(d) Slope field IV

Figure 1: Slope fields for Problem 3

4. Use slope field labeled IV to sketch the graphs of the solutions that satisfy the given initial conditions

$$y(0) = -1, \quad y(0) = 0, \quad y(0) = 1.$$

5. Sketch the slope field of the differential equation. Then use it to sketch a solution curve that passes through the given point

   (a) $y' = y - 2x$, $(1, 0)$

   (b) $y' = xy - x^2$, $(0, 1)$

6. Use Eulers method with step size 0.5 to compute the approximate y-values, y_1, y_2, y_3, and y_4 of the solution of the initial-value problem $y' = y - 2x$, $y(1) = 0$.