MA 123 Elementary Calculus and its Applications
Paper Homework 7-10
Due: 21 July 2008

Instructions

Answer the question in each story. Show your work and explain your solution in *English* – you must have lots of *words* written down for each problem to recieve credit. This means saying what your are doing at each step and why you are doing it. (This is for your benifit. Explaining in writing how you solved the problem forces you to think about the process and admit to yourself when you don't understand what you are doing.)

7. **A Piston** Consider an (idealized) piston that moves up and down in a cylindrical sleeve so that the air column inside has a fixed mass of $1kg$ compressed into a cylinder with a variable height h and radius $.5m$. The height h of the air column starts at $1.1m$ and varies between that and $0.1m$ in such a way that

$$\frac{dh}{dt} = -\frac{\pi}{2}\sin\left(\pi t s^{-1}\right)\frac{m}{s}$$

where t is the time elapsed in seconds. (The s^{-1} is a unit needed to make the units work our right in this formula. The unit s^{-1} is also called a hertz, with symbol Hz. This may be familiar from processor clock speeds, sound frequencies, or radio frequencies.) The density of the air is

$$\rho = \frac{M}{V}$$

meaured in kg/m^3, where $M = 1kg$ is the mass of the air column and V is the volume of the air column. (ρ is the lowercase greek letter "rho".)

Question: At what rate is the denisty of the air is increasing when the height of the air column is $0.25m$ and $t = 3.25318331111$. Give an approximate answer rounded to the nearest $0.1\frac{kg}{m^3 s}$ (neqarest tenth of a kilogram per cubic meter per second).

Warning: You must be sure that your calculator is in "radians" mode. Hit the MODE button and make sure that "radians" and not "degrees" is selected.

8. **Book in a Box** Sally is making a custom sized rectangular scrapbook to give her sister as a gift. She wants to put the book in an elliptical gift box. The box is 8 inches wide and 12 inches long. The graph of the equation

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

is an ellipse centered at the origin that has length $2a$ and width $2b$.

Question: What are the dimensions of the largest rectangular scrap book that Sally can fit in her elliptical box and what its area? (Don't worry the third dimension. Give exact dimension, not approximations.)

HINT: An ellipse (given by the kind of equation above) is symmetric about the x- and y-axes. So you can work with just one quarter of the ellipse and one quarter of the rectangle. Just remember to give the dimensions of the whole book at the end and not just a quarter of it.

9. **Falling Object** The velocity (speed = |velocity|) in feet per second of an object in free fall (free fall means the only force acting on the object is gravity) on Earth t seconds after being released from a height with an initial velocity (positive velocity means up) of of v_0 feet per second is $v = -32t + v_0$ feet per second.

   Question: How far will an object fall in one minute if it is released with no itial veloicity (it is just dropped)? Give either an exact answer (prefered) or an approximate answer that is no more than a half a mile (2640 feet) off from the correct answer (use at least 22 subintervals).

   HINT: Recall that the distance traveled by an object with speed $s(t)$ from time $t = a$ to $t = b$ is the area under the graph of $y = s(t)$ and above the t-axis between $t = a$ and $t = b$.

10. **Baloon** A Baloon has the shape of a cylinder with hemispherical caps of the same radius as the cylinder. The cylindrical part has a height equal to its diameter. The baloon maintains this shape as it grows. I want to inflate the baloon so that total length of the ballon increases at a constant rate of 2 inch per second.

    Question: At what rate should pump air into the balloon? Descbribe the appropriate rate of air as a function of time where the balloon has zero volume at time $t = 0$.

    (Recall that the volume of a sphere with radius r is $4/3\pi r^3$.)