Exam 1 Review - MA 201, Fall 2009.
Example Problem Solutions

1. Let A and B be sets. Give a verbal and mathematical definition of $A \cup B$, $A \cap B$, $\overline{A}$.
   $A \cup B$ is the set of elements that are in A or B or both.
   $A \cup B = \{x | x \in A \text{ or } x \in B\}$.
   $A \cap B$ is the set of elements that are in both A and B.
   $A \cap B = \{x | x \in A \text{ and } x \in B\}$.
   $\overline{A}$ is the set of elements that are not in A. $\overline{A} = \{x | x \notin A\}$.

2. Let the universe $U = \{x | x = 2n, n \leq 10, \text{ and } n \text{ is a whole number}\}$. Write U in list notation. If $A = \{2, 4, 6, 8\}$, what is $\overline{A}$?
   $U = \{0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$
   $\overline{A} = \{0, 10, 12, 14, 16, 18, 20\}$

3. Find a one-to-one correspondence between the set of even whole numbers and the set of odd whole numbers.
   Note that every even whole number can be written as $2n$ for some whole number n. That is,
   $$0 = 2 \cdot 0, 2 = 2 \cdot 1, 4 = 2 \cdot 2, 6 = 2 \cdot 3, \ldots$$
   Every odd number can be written as $2n + 1$ for some whole number n. That is,
   $$1 = 2 \cdot 0 + 1, 3 = 2 \cdot 1 + 1, 5 = 2 \cdot 2 + 1, 7 = 2 \cdot 3 + 1, \ldots$$
   Thus, we can map $2n$ to $2n + 1$. That is, $0 \to 1, 2 \to 3, 4 \to 5$, etc. We see that this is a one-to-one correspondence because every even number is mapped to exactly one odd number, and every odd number has an even number mapped to it.

4. If two sets are equal, must they be equivalent? If true, explain why. If not, give a counterexample.
   Yes, they must be equivalent. Mathematically, we can find a one-to-one correspondence between a set and itself – namely mapping every element to itself. Intuitively, however, we know that a set and itself have the same number of elements making them equivalent.

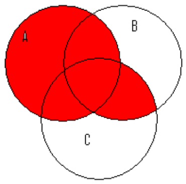
5. If two sets are equivalent, must they be equal? If true, explain why. If not, give a counterexample.
   No, they need not be equal. For example $\{a, b, c\}$ and $\{1, 2, 3\}$ are equivalent (they have the same number of elements) but they are not equal.

6. Your classmate claims that $\emptyset = \{0\}$. Explain why he/she is incorrect and how you would address his/her misunderstanding.
   Your classmate is incorrect because $\emptyset$ represents a set with no elements while $\{0\}$ is a set with one element – the element 0. You would need to mention that, while the number zero represents a set that is empty, zero itself is a number. Therefore, $\{0\}$ is a set containing a number while $\emptyset = \{\}$ does not contain anything.

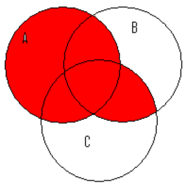
7. Provide an example of two sets which are disjoint. Provide an example of two sets where one is a subset of the other.

$\{1,2,3\}$ and $\{4,5,6,7\}$ are disjoint since they have no elements in common.
$\{1,2,3\}$ is a subset of $\{1,2,3,4,5\}$ because every element in the first set is also in the second.

8. Use Venn diagrams (and be able to explain your picture in words) to show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
$A \cup (B \cap C) =$

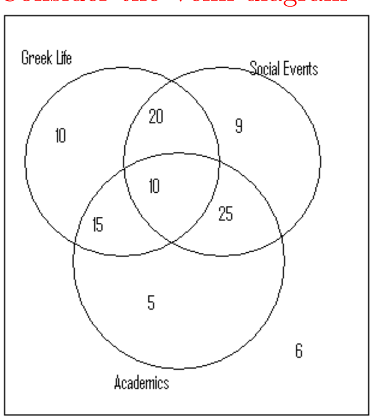


and $(A \cup B) \cap (A \cup C) =$



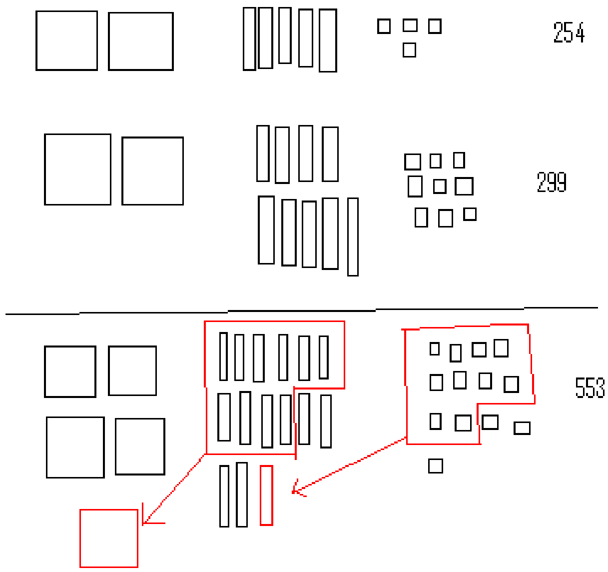
. Since these diagrams are the same, the associated sets must be the same.

9. There are 100 senators on student government and there are three optional committees: academics, Greek life, and social events. Suppose 55 senators are on academics, 55 on Greek life, and 64 on social events. Also suppose that 25 senators serve on both academics and Greek life, 35 on both academics and social events, and 30 on both Greek life and social events. There are 10 senators on all three. How many senators are on more than one committee and how many are on no committee? You must show your work (hint: Venn diagrams!)
Consider the Venn diagram



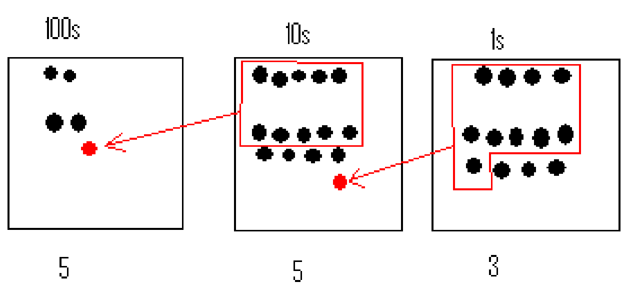
From this, we see that there are $20+10+15+25 = 70$ senators on more than one committee. There are $100-(10+20+9+15+10+25+5) = 6$ senators who are on no committee.

10. Demonstrate $254+299$ using mats, strips, and units. Indicated all exchanges made.



We exchanged 10 units for 1 strips, and then 10 strips for 1 mat.

11. Demonstrate 254 + 299 using place cards and the final algorithm. Indicate all exchanges made.
Using place cards, we have



The final algorithm:

$$\begin{array}{cccc} & 1 & 1 & \\ & 2 & 5 & 4 \\ + & 2 & 9 & 9 \\ \hline & 5 & 5 & 3 \end{array}$$

The exchanges are the same as in the previous problem.

12. Explain the missing factor model of division. Use this model to explain why $a \div 0$ is undefined.
See pages 128 and 130 in the textbook.

13. (a) Convert 42 to base five.

$42 \div 25 = 1R17$
$17 \div 5 = 3R2$
$2 \div 1 = 2R0$
Thus, $42 = 132_{five}$

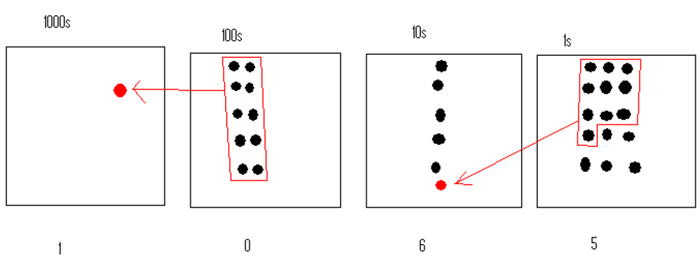
(b) Convert 123_{five} to base 10.
$123_{five} = 1(25) + 2(5) + 3(1) = 36.$

(c) Compute $142_{five} + 234_{five}$.
Using the instructional algorithm:

$$\begin{array}{cccc} & 1 & 4 & 2 \\ + & 2 & 3 & 4 \\ \hline & & 1 & 1 \\ & 1 & 2 & 0 \\ + & 3 & 0 & 0 \\ \hline & 4 & 3 & 1 \end{array}$$

So the answer is 431_{five}.

14. Compute $5 \cdot 213$ using place cards. Indicate all exchanges that you make.



We exchanged ten ones for one ten and ten hundreds for one thousand.

15. Compute $17 \cdot 233$ using expanded notation and the final algorithm.
Expanded notation:

$$\begin{aligned} 17 \cdot 233 &= (10+7) \cdot 233 \\ &= 10(233) + 7(233) \\ &= 10(200+30+3) + 7(200+30+3) \\ &= 10(200) + 10(30) + 10(3) + 7(200) + 7(30) + 7(3) \\ &= 2000 + 300 + 30 + 1400 + 210 + 21 \\ &= 3961. \end{aligned}$$

Final Algorithm:

$$\begin{array}{rrrrr}
 & & 2 & 2 & \\
 & & 2 & 3 & 3 \\
\times & & & 1 & 7 \\
\hline
 & 1 & 6 & 3 & 1 \\
+ & 2 & 3 & 3 & 0 \\
\hline
 & 3 & 9 & 6 & 1
\end{array}$$

16. Use the scaffold method to compute $13511 \div 302$.

$$\begin{array}{r|rrrrrl}
 & & & & 4 & 4 & R223 \\
\cline{5-6}
 & & & & & 4 & \\
 & & & & 4 & 0 & \\
\cline{2-6}
302 & 1 & 3 & 5 & 1 & 1 & \\
- & 1 & 2 & 0 & 8 & 0 & \\
\cline{2-6}
 & & 1 & 4 & 3 & 1 & \\
 & - & 1 & 2 & 0 & 8 & \\
\cline{3-6}
 & & & 2 & 2 & 3 &
\end{array}$$

Note: This list of problems is not exhaustive. Make sure you study your notes and homework as well. Please come see me if you have any questions!