

MA201 Test 2 - 11/3/09

Name: _____

Section: _____

Directions:

Please print your name clearly. This is a 75 minute exam and is worth 15% of your final grade. There are 100 points possible. Calculators may not be used on this exam. Answers without work will receive little or no credit. Good luck!

Problem	Points possible	Points earned
1	8	
2	10	
3	5	
4	16	
5	15	
6	5	
7	5	
8	15	
9	5	
10	6	
11	2	
12	8	
Total	100	
+ Bonus		
Final Score		

1. (a) (4 pts.) State George Polya's four principles for problem solving.

(b) (4 pts.) Using one or two sentences for each, explain what these principles mean.

2. Let a and b be mathematical statements.

(a) (5 pts.) Suppose that the statement “If a then b ” is true. Suppose additionally that b is NOT true. What, if anything, can you conclude? Illustrate this with an example.

(b) (5 pts.) Suppose that the statement “If a then b ” is true. Suppose additionally that a is NOT true. What, if anything, can you conclude? Illustrate this with an example.

3. (5 pts.) Use Gauss’s trick to find the sum $1 + 2 + 3 + 4 + \cdots + (n - 1) + n$. You must show your work.

4. Consider the sequence 1, 5, 9, 13, ...

(a) (4 pts.) List the next 4 terms in this sequence.

(b) (4 pts.) What is the 10th term?

(c) (4 pts.) What is the 100th term?

(d) (4 pts.) What is the n th term?

5. (a) (5 pts.) Explain the pigeonhole principle.

(b) (5 pts.) You have a classroom with 15 boys and 15 girls. When forming a group, how many students must you select at random to ensure that you EITHER have at least two boys OR at least two girls? Explain.

(c) (5 pts.) How many people do you need in a room to ensure that at least two people were born on the same day of the week? Explain.

6. (5 pts.) Sue can work a puzzle in 2 hours. Patty can work the same puzzle in 3 hours. How long would it take the two to work the puzzle together? Make sure you show your work.

7. (5 pts.) Use the Euclidean Algorithm to find $\text{GCD}(1176, 252)$.

8. (a) (5 pts.) Find the prime factorization of 1176 by using a factor tree.

(b) (5 pts.) Suppose $252 = 2^2 \cdot 3^2 \cdot 7$. Use the prime factorizations to calculate $\text{GCD}(1176, 252)$.

(c) (5 pts.) Use the prime factorizations to calculate $\text{LCM}(1176, 252)$.

9. (5 pts.) Let $23a, 5a7, a13$ be a 9 digit number where the a represents a few missing digits. Explain why this number is divisible by three, regardless of the choice of a .

10. (6 pts.) List all factors of the number $n = 23 \cdot 31 \cdot 59^2$ (Note that this is a prime factorization).

11. Are the following statements true or false?

(a) (1 pt.) 1 is prime.

(b) (1 pt.) 0 is neither even nor odd.

12. (8 pts.) Which of the following divide the number 22355080? 2,3,4,5,6,9,10,11. Explain why or why not for each.

Bonus:

13. (5 pts.) Prove why the divisibility test for 3 works.

14. (5 pts.) Explain why there are infinitely many primes.