1. Use Euler's trick to find the sum $1 + 2 + 3 + 4 + \cdots + 49 + 50$.

$$\begin{array}{rcccccccccc} s & = & 1 & + & 2 & + & \ldots & + & 49 & + & 50 \\ s & = & 50 & + & 49 & + & \ldots & + & 2 & + & 1 \\ \hline 2s & = & 51 & + & 51 & + & \ldots & + & 51 & + & 51 \end{array}$$

Thus, $2s = 50 \cdot 51$. Therefore,

$$s = \frac{50 \cdot 51}{2}.$$

2. Consider the sequence $1, 4, 7, 10, \ldots$

   (a) List the next 3 terms in this sequence.
   13,16,19

   (b) What is the 10th term?
   The first term has a 1 plus no threes. The second has 1 plus 1 three. The third has 1 plus 2 threes. Following this pattern, the tenth should have a 1 plus 9 threes. That is $1 + 9 \cdot 3 = 28$

   (c) What is the 100th term?
   $1 + 99 \cdot 3$

   (d) What is the nth term?
   $1 + 3(n - 1)$

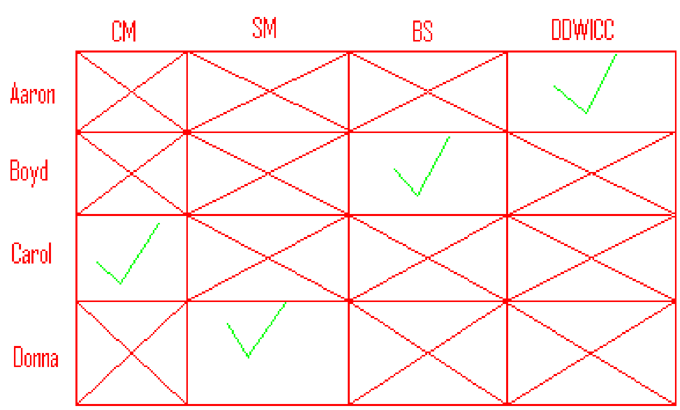
3. One day, Aaron, Boyd, Carol, and Donna went to an ice cream shop. They ordered a chocolate malt, a strawberry milkshake, a banana split, and a double-dip walnut ice cream cone. Given the following information, who had which treat?

   (a) Both boys dislike chocolate.

   (b) Boyd is allergic to nuts.

   (c) Carol bought a malt and a milkshake for Donna and herself.

   (d) Donna shared her treat with Boyd.



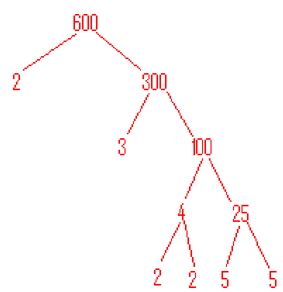
4. In my desk, I have 6 blue pens, 7 black pens, and 3 red pens. How many pens must I remove at random to ensure that I have

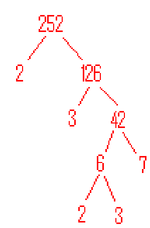
   (a) one red pen? Explain.
   14. It's possible that all 6 blue pens and all 7 black pens could be removed at random before a red pen is removed.

   (b) two of the same color pen? Explain.
   4. The worst case is if I remove one pen of each color (3). However, in this case the fourth pen will be the same color as another. This is an application of the pigeonhole principle where the pen colors are the pigeonholes and the pens are the pigeons.

   (c) one blue and one black pen? Explain.
   11. All 7 black pens and all 3 red pens could be removed first. However, in this case, the eleventh pen must be blue.

5. (a) Draw factor trees for 600 and for 252.





   So $600 = 2^3 \cdot 3 \cdot 5^2$ and $252 = 2^2 \cdot 3^2 \cdot 7$.

   (b) What is GCD(600, 252)? Explain how you got this.
   We know that the greatest common divisor of 600 and 252 will have as many prime factors as possible in common. From part a, we see that each has 2 twos and one 3. They share no other prime factors. Therefore, GCD(600,252)$= 2^2 \cdot 3 = 12$.

   (c) What is LCM(600, 252)? Explain how you got this.
   The least common multiple must contain every prime factor of each number. Therefore, we must have 3 twos, 2 threes, 2 fives, and 1 seven. Thus, LCM(600,252)$= 2^3 \cdot 3^2 \cdot 5^2 \cdot 7 = 12600$.

6. Use the Euclidean Algorithm to find GCD(4004,1092).
We know that 4004 and 1092 have the same GCD as 1092 and r where r is the remainder when we divide 4004 by 1092. Thus, we calculate (using algorithm of your choice) that $4004 = 3 \cdot 1092 + 728$. Therefore, r is 728. Repeating this argument, we know that 1092 and 728 have the same GCD as 728 and s where s is the remainder when we divide 1092 by 728. We see that $1092 = 1 \cdot 728 + 364$. So $s = 364$. Repeating this argument again, we see that 728 and 364 have the same GCD as 364 and t where t is the remainder when we divide 728 and 364. We see that $728 = 2 \cdot 364 + 0$. So $t = 0$. However, it's clear that GCD(364,0)=364. So GCD(4004,1092)=364.

7. It takes a teacher 6 hours to grade exams. It would take a grading assistant 8 hours to grade the same exams. How long will it take them to grade the exams together?

Let t be the number of hours it takes both to grade the exams. In t hours, the teacher can grade $\frac{t}{6}$ of the exams. In t hours, the teaching assistant can grade $\frac{t}{8}$ of the exams. Therefore, in t hours, they can grade $\frac{t}{6} + \frac{t}{8}$ of the exams which is all of them. Therefore, we get the equation

$$\frac{t}{6} + \frac{t}{8} = 1.$$

Solving for t, we see

$$\frac{7t}{24} = 1,$$

which implies that $t = \frac{24}{7}$

8. Which of the following divide the number 4437048? 2,3,4,5,6,9,10,11. Explain why or why not for each.

Number	Divisible?	Reason
2	Yes	The last digit is an 8 which is divisible by 2
3	Yes	4+4+3+7+4+8=30 which is divisible by 3.
4	Yes	The number created by the last two digits, 48, is divisible by 4.
5	No	The last digit is an 8 which is not divisible by 5.
6	Yes	The number is divisible by 2 and 3.
9	No	4+4+3+7+4+8=30 which is not divisible by 9.
10	No	The last digit is an 8 which is not divisible by 10.
11	Yes	4-4+3-7+0-4+8 =0 which is divisible by 11.

9. Use deductive reasoning to prove the following statement: "If n is an odd number, then $n+n$ is even."
Let n be an odd number. Then $n = 2k+1$ for some whole number k. Thus,

$$\begin{aligned} n+n &= 2k+1+2k+1 \\ &= 4k+2 \\ &= 2(2k+1) \end{aligned}$$

which is even (because it is divisible by two).

10. If you were to use indirect reasoning to prove the previous statement, what statement would you be trying to prove?
If $n+n$ is odd, then n is odd.

11. Suppose to know that "If a then b" is true. Suppose additionally that you know b is true. What can you conclude?
You can conclude nothing. For example, we know, "If I touch a hot stove, then I get burned," is a true statement. If I also tell you that I got burned, you CANNOT conclude that I touched a hot stove. I could have spilled hot water on myself or stayed in the sun too long!

12. Determine whether the following statements are true or false.

    (a) 1 is a prime number.
    False, one is neither prime nor composite

(b) If a number n is divisible by 3 and by 6, then n is divisible by 18.
False. For example, 12 is divisible by 3 and 6, but 12 is NOT divisible by 18.

(c) For natural numbers a, b, d, if d divides a and b, then d divides $a - b$.
True.

(d) 213651 is divisible by 9.
True. Since 2+1+3+6+5+1=18 and 18 is divisible by 9 then so is 213651.

(e) If pigs can fly, then $2 + 2 = 5$.
True. If the hypothesis of a conditional statement is false, we are allowed to conclude anything we want!

13. List all of the factors of $n = 2^2 \cdot 3 \cdot 5^2$.

$2^0 \cdot 3^0 \cdot 5^0 = 1$	$2^1 \cdot 3^0 \cdot 5^0 = 2$	$2^2 \cdot 3^0 \cdot 5^0 = 4$
$2^0 \cdot 3^0 \cdot 5^1 = 5$	$2^1 \cdot 3^0 \cdot 5^1 = 10$	$2^2 \cdot 3^0 \cdot 5^1 = 20$
$2^0 \cdot 3^0 \cdot 5^2 = 25$	$2^1 \cdot 3^0 \cdot 5^2 = 50$	$2^2 \cdot 3^0 \cdot 5^2 = 100$
$2^0 \cdot 3^1 \cdot 5^0 = 3$	$2^1 \cdot 3^1 \cdot 5^0 = 6$	$2^2 \cdot 3^1 \cdot 5^0 = 12$
$2^0 \cdot 3^1 \cdot 5^1 = 15$	$2^1 \cdot 3^1 \cdot 5^1 = 30$	$2^2 \cdot 3^1 \cdot 5^1 = 60$
$2^0 \cdot 3^1 \cdot 5^2 = 75$	$2^1 \cdot 3^1 \cdot 5^2 = 150$	$2^2 \cdot 3^1 \cdot 5^2 = 300$

14. Explain why the number abc, abc is always divisible by 11.
Using the divisibility test for eleven, we take the alternating sum of the digits. That is

$$-a + b - c + a - b + c = 0,$$

regardless of what a, b, and c are. Since 0 is divisible by eleven, so is the number abc, abc.

15. Suppose a and b are divisible by k. Prove that $a + b$ is divisible by k.
Since a is divisible by k, we can write $a = k \cdot i$ for some integer i. Likewise, we can write $b = k \cdot j$ for some integer j. Therefore,

$$\begin{aligned} a + b &= k \cdot i + k \cdot j \\ &= k \cdot (i + j). \end{aligned}$$

Since we have written $a + b$ as k times the integer $(i + j)$, $a + b$ must be divisible by k.

16. Find a value of the ones digit a such that the number $235, 47a$ is divisible by

(a) two.
a could be 0,2,4,6, or 8.

(b) three.
$2 + 3 + 5 + 4 + 7 + a = 21 + a$. We need this to be divisible by 3. Therefore, we could have $a = 0$ (because 21 is divisible by 3), $a = 3$ (because 24 is divisible by 3), $a = 6$ (because 27 is divisible by 3), or $a = 9$ (because 30 is divisible by 3).

(c) four.
We need the two digit number $7a$ divisible by 4. Therefore, we could have $a = 2$ (because 72 is divisible by 4), or $a = 6$ (because 76 is divisible by 4).

(d) five.
a could be 0 or 5.

(e) nine.
$2 + 3 + 5 + 4 + 7 + a = 21 + a$. We need this to be divisible by 9. Therefore, we must have $a = 6$ (because 27 is divisible by 9).

(f) ten.
We must have $a = 0$.

(g) eleven.
$-2 + 3 - 5 + 4 - 7 + a = -7 + a$. We need this to be divisible by 11. Therefore, we must have $a = 7$ (because 0 is divisible by 11).