MA201 Test 3 key

1. (5 pts) Explain the density property for rational numbers.
   The rational numbers are dense meaning that between any two rational numbers is another rational number.

2. (5 pts) Find three rational numbers between $\frac{1}{3}$ and $\frac{4}{9}$.
   $\frac{1}{3} = \frac{12}{36}$ and $\frac{4}{9} = \frac{16}{36}$. Thus, we see that $\frac{13}{36}$, $\frac{14}{36}$, and $\frac{15}{36}$ are all between $\frac{1}{3}$ and $\frac{4}{9}$.

3. The length of a day on Jupiter is about 10 hours. The Jupiter children were doing arithmetic on their 10 hour clock. Help them solve the following problems.

   (a) (3 pts) $7 +_{10} 5 = 2$

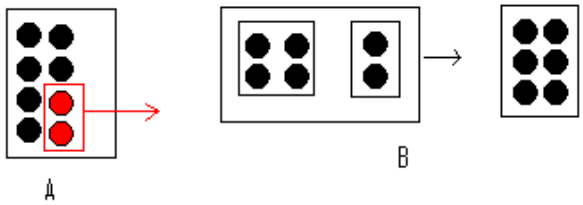
   (b) (3 pts) $3 -_{10} 8 = 5$

   (c) (3 pts) $5 \times_{10} 5 = 5$

   (d) (3 pts) What is the additive inverse of 7 in 10 hour clock arithmetic?
   3

4. (a) (5 pts) Illustrate $4 - (-2)$ using color counters. Explain your picture using complete sentences.

   (b) (5 pts) Illustrate $4 + 2$ using color counters. Explain your picture using complete sentences.



   Red will indicate -1 while black indicates $+1$. In part (a) we had to represent the number four using two red counters. We do this by starting with 4 black counters and then adding two red/black pairs. Now we remove 2 red counters and are left with 6 black.
   For part (b), we represent 4 with black counters and 2 with black counters then combine them to get 6 black counters.

   (c) (5 pts) Explain, using complete sentences, why $a - (-b) = a + b$.
   We can see from the previous two problems that we must represent the number a using b red counters. In order to do this we will start with a black counters and then add b red/black pairs. After removing the b red counters, we are left with $a + b$ black counters.

5. Compute the following:

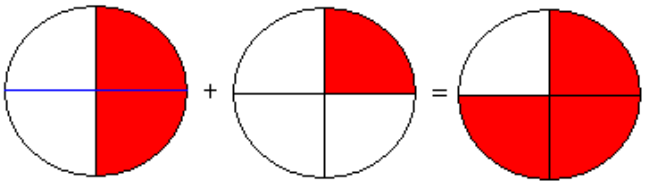
   (a) (3 pts) $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$.

   (b) (3 pts) $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.

   (c) (3 pts) $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$.

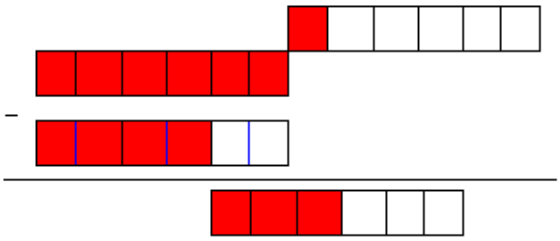
6. For the following, explain how to compute the problem using the indicated conceptual model.

(a) (5 pts) Model $\frac{1}{2} + \frac{1}{4}$ using colored regions.



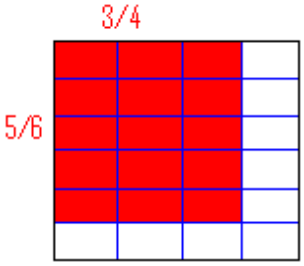
We start with a representation of $\frac{1}{2}$ and $\frac{1}{4}$. Since the pieces of the first circle are twice as big, we break each piece into two pieces. Now, since all the pieces are the same size, we see we have 3 pieces out of 4. So the answer is $\frac{3}{4}$.

(b) (5 pts) Model $\frac{7}{6} - \frac{2}{3}$ using fraction strips.



We represent $\frac{7}{6}$ and $\frac{2}{3}$ with strips. We notice that the pieces of the lower strip are twice as big so we break each of the pieces in half. Now since the pieces are all the same size, we can remove the ones we need to. We can see that we are left with 3 of a possible 6 pieces. So the answer is $\frac{3}{6}$.

(c) (5 pts) Model $\frac{5}{6} \cdot \frac{3}{4}$ using a rectangular array.



We break our row into 6 pieces and consider 5 of them. We break our column into 4 pieces and consider 3 of them. The area covered by this is 15 pieces out of a possible 24. So the answer is $\frac{15}{24}$.

7. Write a word problem that models each of the following.

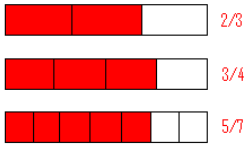
(a) (5 pts) $\frac{1}{2} \times \frac{3}{4}$
A recipe for chili calls for $\frac{3}{4}$ cup of beans. You plan on making only half a recipe of chili. How many cups of beans do you need?

(b) (5 pts) $3 \div \frac{1}{2}$
Jimmy has three candy bars. If he wants to give each of his friends half a candy bar, how many friends can he feed?

8. (5 pts) A student in your class claims the following:

$$\frac{2}{3} + \frac{3}{4} = \frac{5}{7}.$$

What is the student's mistake and how would you address his/her misunderstanding? Make sure you use a conceptual model.



The student added straight across the fractions. I would show the student the above pictures of fraction strips. It becomes clear that if put the $\frac{2}{3}$ strip together with the $\frac{3}{4}$ strip, it is bigger than the $\frac{5}{7}$ strip.

9. (a) (3 pts) Write $\frac{90}{135}$ in simplest form.
$\frac{90}{135} = \frac{2 \cdot 45}{3 \cdot 45} = \frac{2}{3}$

(b) (3 pts) Write $\frac{11}{4}$ as a mixed number.
$\frac{11}{4} = 2\frac{3}{4}$.

(c) (3 pts) Write $3\frac{2}{5}$ as a fraction.
$3\frac{2}{5} = \frac{17}{5}$.

(d) (3 pts) Compute $1\frac{3}{4} \div \frac{1}{2}$.
$1\frac{3}{4} \div \frac{1}{2} = \frac{7}{4} \times \frac{2}{1} = \frac{14}{4}$.

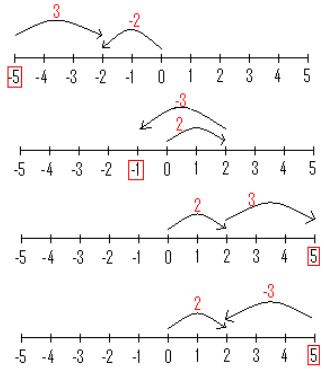
10. Use a number line to model the following problems (make sure you label everything appropriately).

(a) (3 pts) $-2 - 3$

(b) (3 pts) $2 + (-3)$

(c) (3 pts) $2 + 3$

(d) (3 pts) $2 - (-3)$



Bonus:

11. (5 pts) What is $|3 - \pi|$?
$\pi - 3$

12. (5 pts) Write $.\overline{27}$ as a fraction.
$\frac{27}{99}$