SOLUTIONS

Math 111 First Voting Worksheet

A committee of twenty people is trying to decide which of four departments is going to get funding. The four departments are Art (A), English (E), History (H), and Political Science (P). Their preference ballots look as follows.

A	P	A	E	P	E	P	A	A	E	P	P	P	E	E	E	E	P	P	A
H	A	H	A	E	A	A	H	H	A	E	E	A	A	A	A	A	E	A	H
E	E	E	P	A	P	E	E	E	P	A	A	E	P	P	P	P	A	H	E
P	H	P	H	H	H	H	P	P	H	H	H	H	H	H	H	H	H	E	P

1. Create a preference schedule based on the above ballots:

# of voters	5	3	7	4	1
1st	A	P	E	P	P
2nd	H	A	A	E	A
3rd	E	E	P	A	H
4th	P	H	H	H	E

2. Using the plurality method, which choice wins?

Count first-place votes: A 5
E 7
P 4+3+1 = (8)
H 0

Winner is Political Science.

3. Who is the winner using plurality with elimination (also called "instant runoff voting")? (show steps clearly!)

Political Science does not have a majority of 1st place votes (with 20 voters, we require 11 for majority.)

Eliminate History: (Fewest 1st place) → next eliminate Art:

5	3	7	4	1
A	P	E	P	P
E	A	A	E	A
P	E	P	A	E

Count 1st place votes:

A 5, P 8, E 7

Still no majority winner.

5	3	7	4	1
E	P	E	P	P
P	E	P	E	E

Count 1st place votes:

E 5+7 = (12)

P 3+4+1 = 8

Winner is English.

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Refill the table here:

	# of voters	5	3	7	4	1
4 Borda points	1st	A	P	E	P	P
3	2nd	H	A	A	E	A
2	3rd	E	E	P	A	H
1 Borda point	4th	P	H	H	H	E

4. **Who is the winner using the Borda count method? (Show the computations to calculate Borda points for each department.)**

A: $5(4) + 3(3) + 7(3) + 4(2) + 1(3) = 61$ (Winner is Art)

H: $5(3) + 3(1) + 7(1) + 4(1) + 1(2) = 31$

P: $5(1) + 3(4) + 7(2) + 4(4) + 1(4) = 51$

E: $5(2) + 3(2) + 7(4) + 4(3) + 1(1) = 57$

error check: each of these should have all of 1, 2, 3, 4

another error check: Total number of Borda points

$61 + 31 + 51 + 57 = 200$

Matches $20\ (1+2+3+4) = 20(10) = 200.$

(20 = # voters)

5. **How many different ballots were possible for this election? (Show the formula)**

There are four candidates, so there are

$4 \cdot 3 \cdot 2 \cdot 1 = 24$ possible ballots.

(The fact that there are only five different columns is a clue that this is made-up data, not a real election.)