

Names:

SOLUTIONS

1. Consider the following election. with 100 votes

a. Who wins using with Plurality with Elimination?
 No Maj., elim McGee; redraw table:

45	35	20	
G	T	G	
T	G	T	

Now **Gibbs wins**
 G: 65
 T: 35

	45	35	20
1 st	Gibbs	Tony	McGee
2 nd	McGee	McGee	Gibbs
3 rd	Tony	Gibbs	Tony

Note: this was just an example.
 Can test IIA by removing any losing candidate.

b. Now we will disqualify one of the losers: who has the most last place votes? Remove that person and find the new winner. Does the winner change from part (a)? **yes**

MOST last place is Tony.

without Tony,	45	35	20
	G	M	M
	M	G	G

McGee wins with 55
 Gibbs has 45

c. Which fairness criterion were we testing in part (b)? Did our test show a violation, or did the test not apply? By elim. a loser, we tested **IIA**.

Since the winner changed, we see Plur. w/ Elim **violates** IIA.

2. In the preference schedule below, suppose A wins the election above using a mystery method.

	?	?	?	?	?	?
1 st	A	A	B	B	C	C
2 nd	B	C	A	C	A	B
3 rd	C	B	C	A	B	A

For A to move up, possible ballots to test monotonicity:
 C or A
 A or C
 B or B

We want to test whether this voting method violates the Monotonicity Criterion by changing ballots from the last column. Which of the following is a valid change to test Monotonicity? (The ballots are listed as 1st, 2nd, 3rd)

- C, A, B
- A, B, C
- C, B, A
- B, A, C
- none of these

3. The plurality method violates the Condorcet criterion. That means (select one)

- a. It is impossible to have a Condorcet candidate in an election using the plurality method
- b. Condorcet candidates always lose with plurality method
- c. Condorcet candidates can lose with the plurality method
- d. Condorcet candidates are never majority candidates

4. Majority candidates are always (select all that are true):

- a. Losers using the Borda Count method
- b. Winners with Pairwise Comparison
- c. Winners with Borda Count method
- d. Losers with the Plurality with Elimination method
- e. Eliminated first in Plurality with Elimination

SOLUTIONS

5. Suppose we have an election with seven candidates, A, B, C, D, E, F, G; and 100 people voting.

a. How many pairwise points would A need to be a Condorcet candidate?

$X-1 = 7-1 = 6$ (A must win over every other candidate.)

b. How many different possible ballots are there? (formula and answer)

$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$

c. If we used the Borda count method, what would the total of the Borda points be? (formula)

$100(7+6+5+4+3+2+1) = 100(28) = 2800$

6. The following election uses an unknown method. You will use this election to test this mystery method against one (or more) of the fairness criteria.

a. Which candidate got over 50% of the first place votes? What kind of candidate is this?

1	2	2	4
B	A	C	B
A	C	B	C
C	B	A	A

B is majority candidate, first place votes (with 5 of the 9)

b. By asking this question, which fairness criteria are we testing? (Can we test more than one?)

Majority Criterion.

Since Maj. is Condorcet, also testing Condorcet.

c. Suppose our voting method selects A as the winner. Does this imply our voting method violates a fairness criterion, satisfies a fairness criterion, or that the test doesn't apply?

Violates: B is maj. cand., and B loses. (also violates Condorcet)

d. Suppose our voting method selects B as the winner. Does this imply our voting method violates a fairness criterion, satisfies a fairness criterion, or that the test doesn't apply?

Inconclusive: only violate if there is a maj. candidate and they lose (one example cannot show it satisfies by itself.)

7. Construct an example of an election between Rory, Amy and Clara with 60 voters in which Clara wins by plurality, but Amy wins using plurality with elimination.

21	20	19
C	A	R
A	R	A
R	C	C

Clara has the most first-place votes (plur. winner).
 Plur w/elim: No maj. cand. → eliminate R: Amy wins plur. w/elim.

21	20	19
C	A	A
A	C	C

8. Construct an example of an election between McDonald's (M), Burger King (B) and Arby's (A) with 60 voters where McDonalds is the winner using Borda count, Burger King wins using plurality, but Burger King is NOT a majority candidate.

check all 3 conditions!

21	20	19
B	M	A
M	A	M
A	B	B

careful that M doesn't win plurality: all candidates must get some 1st place votes

Borda: $B = 21(3) + 20(1) + 19(1) = 102$
 $M = 21(2) + 20(3) + 19(2) = 140$
 $A = 21(1) + 20(2) + 19(3) = 118$