Math 111 – 012
November 10, 2016

Compensation Practice
Exam 3 Review

Names: SOLUTIONS

I. Bert, Ernie and Grover have won a ride in a rocket, but only one of them will get to ride. They decide to give the ride to the highest bidder, and that whoever takes the ride should compensate the others equitably. Bert thinks the ride is worth \$900; Ernie bids \$630, and Grover (who is a little bit afraid of heights) bids \$270.

1. Find the **fair shares** for each person.

Bert: $\frac{900}{3} = \$300$ Ernie: $\frac{630}{3} = \$210$ Grover: $\frac{270}{3} = \$90$

2. **Who** could possibly get the ride if we require a **fair** settlement? Explain briefly. (Hint: begin by computing the average of the bids.)

Average bid: $\frac{960+630+270}{3} = \$600$. Both Bert and Ernie bid at least \$600, so either of them could receive the ride for a fair settlement

3. Give ***any*** example of an **envy-free** settlement. (Hint: find the range of possible payments, and then pick one.)

Envy-free requires that Bert (the highest bidder) gets the ride.

Ernie's fair share of \$210 ≤ possible payment ≤ Bert's fair share of \$300

For example: Bert gets the ride and pays \$500, Ernie and Grover each receive \$250.

II. Kermit and Miss Piggy are getting a divorce! They have several assets to divide which they can't agree on, and decide to use the **Adjusted Winner method** to divide them fairly. Show the steps and give the final settlement.

		Kermit	Piggy
A.	framed artwork	15	(20)
B.	Rolls Royce	(30)	10
C.	NY apartment	(45)	30
D.	Paris apartment	10	(40)

Initially, Kermit has 75, Piggy has 60, so Kermit is the giver.

Point ratios for Kermit's items: (B) Roll's Royce $\frac{30}{10} = 3$ (C) NY apt $\frac{45}{30} = 1.5$

Smallest (closest to 1) is the NY apt. Kermit cannot transfer the entire apartment (would tip the balance to Piggy), so they will share the apartment.

Let p be the % of the NY apt Kermit keeps.

Transfer equation:

Kermit's other points + p(Kermit's value of NY apt) = Piggy's points + (1−p)(Piggy's value of apt)

$$30 + p(45) = 60 + (1-p)(30)$$

Solve the equation:

$30 + 45p = 60 + 30 - 30p$
$30 + 45p = 90 - 30p$
$-30 + 30p \quad -30 + 30p$
$75p = 60$
$p = \frac{60}{75} = 80\%$

Final settlement

Kermit gets the Rolls Royce and 80% of the NY apt.

Piggy gets the artwork, the Paris apartment, and 20% of the NY apartment.

III. James Bond, Moneypenny, M, and Q have to divide a collection including a poisoned pen, a magnetic-resistant watch, and an underwater jet-pack. They each value the items as shown. **Here is the settlement** they come up with:

	Bond	Moneypenney	M	Q
pen	100	300	400	500
watch	500	500	400	600
jet-pack	900	100	200	800
Total	1500	900	1000	1900
Fair share	375	225	250	475

Suppose that Bond takes the jet-pack and pays $500, M takes the pen and receives $100, Q takes the watch and pays $200, and Moneypenney receives $600.

1. Find x_{Bond}, $x_{\text{Moneypenney}}$, x_{M} and x_{Q}.

$x_{Bond} = 900 - 500 = 400$ $\quad x_M = 400 + 100 = 500$

$x_{Moneypenney} = 600$ $\quad x_Q = 600 - 200 = 400$

2. Is this settlement **fair**? Explain.

No: Q's fairshare is $475, but x_q is only $400.

(it is fair to every other player, since each got their fair share or more; but to be fair overall must be fair to everyone.)

3. How much is Bond's settlement worth to Q? Does Q envy Bond?

Bond's settlement: get jetpack, pay $500

Worth to Q: $800 - 500 = \boxed{\$300}$

Q does not envy Bond because $x_q = 400$, more than 300.

4. Compute the XB ratios for Bond and for M.

Bond: $\frac{x_{Bond}}{b_{Bond}} = \frac{400}{1500} \boxed{= .2667}$ $\quad$ M: $\frac{x_M}{b_M} = \frac{500}{1000} \boxed{= .5}$

IV. More practice

a. For problem I (Bert, Ernie and Grover), find a settlement that is **equitable**. (Hint: use the equitability method). Show steps clearly.

Fair shares: Bert 300, Ernie 210, Grover 90.

First settlement: Bert gets ride, pays $600. Ernie gets 210, Grover gets 90.

Surplus: Total paid - total received = 600 - (210 + 90) = 600 - 300 = $300.

Now divide the surplus. Total of all bids is 900 + 630 + 270 = 1800.

Bert's share = $300\left(\frac{900}{1800}\right) = \$150$

Ernies share = $300\left(\frac{630}{1800}\right) = \$105$

Grover's share = $300\left(\frac{270}{1800}\right) = 45$

(check: these add to $300.)

FINAL settlement: Combine 1st settlement with share of surplus

Bert gets ride, pays $450

Ernie gets $315

Grover gets $135.

(check: total paid = total received.)

b. For problem I part 3 (your envy-free settlement), how would you ***justify*** that your settlement is envy-free? (Explain the process.)

We'd check pairs: since Ernie, Grover got the same amount of cash, they don't envy each other. We make sure Ernie doesn't envy Bert, that Bert doesn't envy Ernie, and the same with Bert and Grover.

c. For problem I part 3, is your envy-free settlement also **equitable**? Explain why or why not. Include calculations. Compute XB ratios: (will vary based on your choice of payment!)

Bert: $\frac{x_{Bert}}{b_{Bert}} = \frac{900-500}{900} = \frac{400}{900} = .44$

Ernie: $\frac{x_{Ernie}}{b_{Ernie}} = \frac{250}{630} = .3968$

Grover: $\frac{x_{Grover}}{b_{Grover}} = \frac{250}{270} = .9259$

[Since XB ratios are not equal, settlement is NOT equitable.]

d. For problem III (James Bond, Moneypenny, M, and Q), find a settlement using Knaster's method. Use your own paper to work out the calculations; give just the **final settlement** here:

Bond gets jetpack, pays $356.25

Q gets pen and watch, pays $456.25

Moneypenny receives $393.75,

M receives $418.75

check: total paid matches total received.