

Names:

SOLUTIONS

- I. Brian, Stewart and Peter work for Acme Incorporated, which is getting some new office supplies this month. The boss (their uncle) decides that the three of them can have the old equipment as long as they agree to share it equally between them. They use **Knaster's method of sealed bids**, with the bids shown below. Round to the nearest penny when needed.

	Brian	Stewart	Peter
cabinet	\$60	\$65	\$80
evil monkey	\$35	\$30	\$40
printer	\$90	\$70	\$50
copier	\$200	\$220	\$200
<i>Total</i>	<i>\$385</i>	<i>\$385</i>	<i>\$370</i>

- How much is a **fair share**, according to Brian? (Find how much the goods are worth to Brian, and then divide by 3, the number of people.) $\frac{385}{3} \approx \$128.33$
- Which goods will Brian get, if any? (Which is he the highest bidder on?) **printer**
- How much cash will Brian pay or receive in the first settlement? (What cash should he receive or pay in order to have EXACTLY his fair share?)
His \$90 printer is not enough to be his fair share, he should **receive** $\$128.33 - \$90 = \$38.33$ in cash.
- How much is a fair share according to Stewart? $\frac{385}{3} \approx \$128.33$
- Which goods will Stewart get, if any? **Copier**
- How much cash will Stewart pay or receive in the first settlement?
His \$220 copier is too much to be his fair share; he should **pay** $\$220 - \$128.33 = \$91.67$ in cash.
- How much is a fair share according to Peter? $\frac{370}{3} \approx \$123.33$
- Which goods will Peter get, if any? **Cabinet and monkey**
- How much cash will Peter pay or receive in the first settlement?
Cabinet and monkey are worth \$120, which is not enough for his fair share; he also receives \$3.33 in cash.
- What is the surplus? (Find the difference between the total amount paid out and the total amount received in the first settlement: lines 3, 6 and 9 above.)
Total paid – total received = $\$91.67 - (\$38.33 + \$3.33) = \50.01 .

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11. Divide the surplus by three and award it to each person. Then write out the **final settlement** below. (You'll have **one sentence for each person**: *Brian receives [items] and pays/receives [cash]; etc.*)

Each will get $\frac{50.01}{3} \approx \16.67 of the surplus.

Final settlement:

Brian receives the **printer** and $38.33 + 16.67 = \$55$ in cash.

Stewart receives the **copier** and **pays** $91.67 - 16.67 = \$75$ in cash.

Peter receives the **cabinet** and the **monkey** and $3.33 + 16.67 = \$20$ in cash.

Note: the total amount paid (\$75) matches the total amount received (\$55 + \$20).

- II. **Knaster's method is not the only way to divide up items.** Suppose that, instead of using Knaster's method, a judge had allotted the items as follows:

Brian gets the printer and copier, pays \$160

Stewart gets the cabinet, and receives \$70,

Peter gets the evil monkey and receives \$90.

Is this settlement *fair*? Why or why not? Show computations and explain. You should have one sentence for each player, justifying why or why not the settlement is fair to them.

(*Hint: it doesn't matter whether or not the items went to the highest bidder. For this type of division, the definition of fair means that each player received at least their fair share in net goods and cash.*)

Solution:

Brian received a \$90 printer and \$200 copier, so he has items worth \$290; he paid \$160 so his settlement is worth $\$290 - \$160 = \$130$. This is more than his fair share (\$128.33), so this IS fair to Brian.

Stewart received a \$65 cabinet and \$70 cash, so his settlement is worth $\$65 + \$70 = \$135$; this is more than his fair share (\$128.33), so this IS fair to Stewart.

Peter received a \$40 monkey and \$90 cash; his settlement is worth \$130; this is more than his fair share (\$123.33), so this IS fair to Peter.

Since all players agree their settlement is fair, this IS a fair settlement.