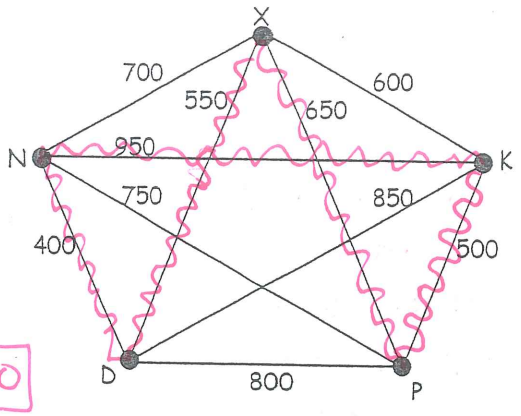
MATH 111

December 6, 2016

HAMILTONIAN CIRCUITS

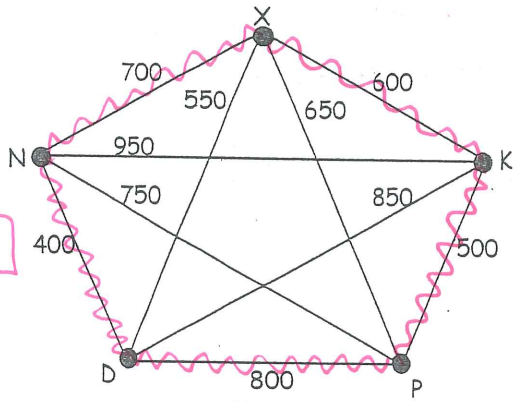
NAME: SOLUTIONS

1. Suppose Marvin, who lives on Planet X, must visit the planets of Krypton, Pern, Dagobah, and Narn. The weights shown in the diagram represent the number of weeks required for him to travel between the planets.



(a) Apply the Nearest Neighbor algorithm with starting vertex K to find a Hamiltonian circuit. Show your circuit on the graph at right. What is the total travel time for this circuit?

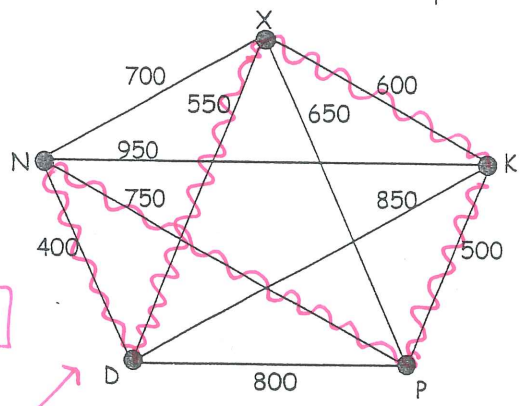
500 + 650 + 550 + 400 + 950 = **3050**



(b) Apply the Nearest Neighbor algorithm with starting vertex D to find a Hamiltonian circuit. Show your circuit on the graph at right. What is the total travel time for this circuit?

400 + 700 + 600 + 500 + 800 = **3000**

(c) Apply the Cheapest Link (Sorted Edges) algorithm to find a Hamiltonian circuit. **Show all steps.** Show your circuit on the graph at right. What is the travel time for this circuit?



400
500
550
600
650 – three at X

700 three at X
750 DONE!
800
850
950

TOTAL: 2800

(d) For the best of the circuits you found in parts (a)–(c), write the circuit as it would be traveled by Marvin, living on Planet X. (list vertices in order)

use answer (c): start/end with X. **XKPNDX** or **XDNPKX**

2. Now suppose that Marvin must travel from Planet X to 13 ~~other~~ planets.

(a) How many edges will the complete graph between these 14 planets have? (show the formula, and the result.)

$\frac{n(n-1)}{2} = \frac{14(14-1)}{2} = \frac{14(13)}{2} = 91$ edges

(b) How many different Hamiltonian circuits are possible for this graph? (show the formula, and the result.)

$\frac{(n-1)!}{2} = \frac{(14-1)!}{2} = \frac{13!}{2} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2}$

= 3113510400 circuits

(c) If we used the Repetitive Nearest Neighbor Algorithm for this graph, how many circuits would we need to check? **14** check Nearest Neighbor circuit from each vertex as starting point.

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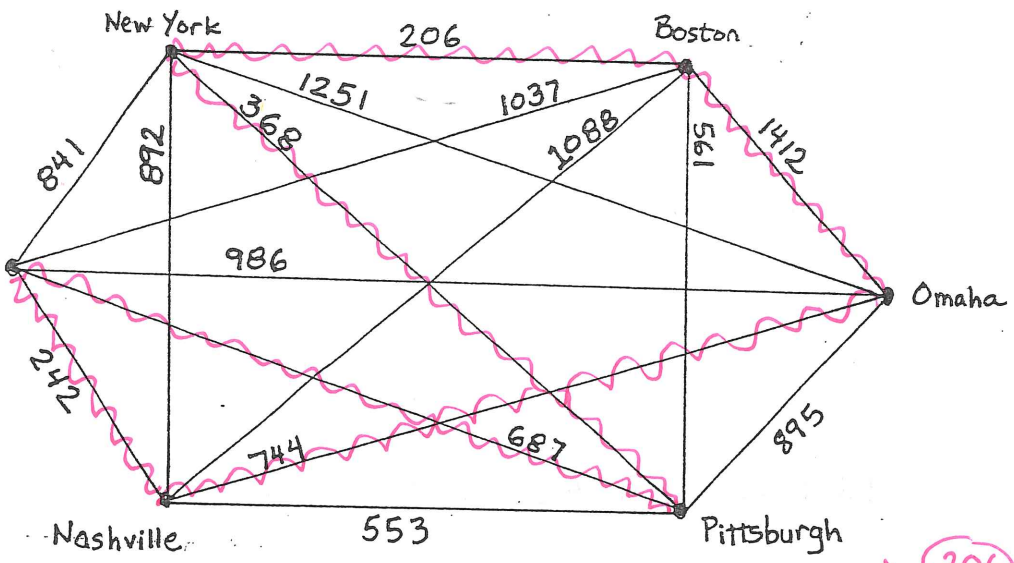
SOLUTIONS

3. Moe must visit all of his Aunts and Uncles, who live in the cities listed below. For each route you find, give the total cost of the Hamilton circuit.

(a) Use the Nearest-Neighbor method to find a low-mileage route starting and ending at Omaha.

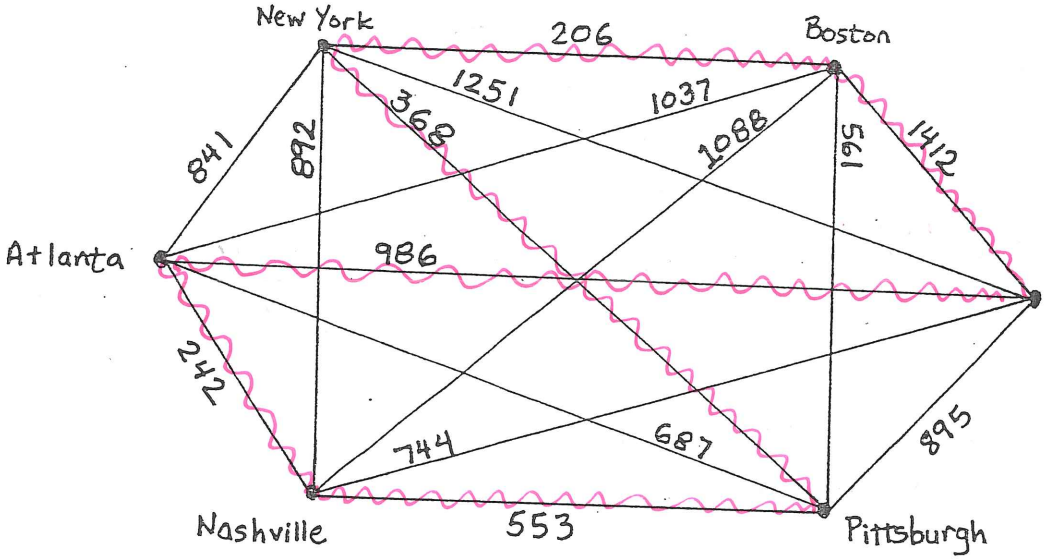
① Om - Nash 744
② Nash - Atl 242
③ Atl - Pitt 687
④ Pitt - NY 368
⑤ NY - Bos 206
⑥ Bos. Om 1412

(Total 3659)



(b) use sorted edges algorithm

206
242
368
553
~~561~~ 3 at Pitt
~~687~~ 3 at Pitt
~~744~~ 3 at Nash
~~841~~ 3 at NY
~~892~~ 3 at NY
~~895~~ 3 at Pitt
986 *
~~1037~~ 3 at Atl
~~1088~~ 3 at Nash
1412



* once you have all but one edge, only one edge possible to finish.

4. How many Hamilton circuits would we need to check in order to find the BEST route for a traveling salesman who must visit 7 cities and then return to his home (in the 8th city)?

$$\frac{(n-1)!}{2} = \frac{(8-1)!}{2} = \frac{7!}{2} = \boxed{2520}$$