Math 111 Fourth Probability Worksheet

Print **all** group member's names here. Circle the name of the group member who turns this in.

SOLUTIONS

A survey of 75 automobiles parked on a university campus lot classified the brands by country of origin and by the type of parking permit (student or faculty/staff).

	American car (A)	European car (E)	Asian car (N)	total
student (S)	25	10	15	50
faculty/staff (F)	9	4	12	25
total	34	14	27	75

1. Suppose we choose a car at random. Find the following probabilities (leave your answer as fractions; do not reduce):

a. $P(S) = \boxed{50/75}$

b. $P(E) = \boxed{14/75}$

c. $P(\bar{N}) = 1 - \frac{27}{75} = \boxed{\frac{48}{75}}$

d. $P(S|A) = \boxed{\frac{25}{34}}$

e. $P(E \cap S) = \boxed{\frac{10}{75}}$

f. $P(E \cap N) = \boxed{0}$

g. $P(E \cup F) = \frac{14}{75} + \frac{25}{75} - \frac{4}{75} = \boxed{\frac{35}{75}}$

h. $P(E \cup N) = \frac{14}{75} + \frac{27}{75} - \frac{0}{75} = \boxed{\frac{41}{75}}$

2. Are the events A and S independent or dependent? Justify your answer with the correct notation and computations. (a) use conditional probabilities; (b) use intersections

(a) Check: Does $P(S|A) = P(S)$? $P(S|A) = \frac{25}{34} \approx .735$, $P(S) = \frac{50}{75} \approx .666$

They do Not match: events are dependent.

(or, could check if $P(A|S) = P(A)$.)

(b) Check: Does $P(A \cap S) = P(A) \cdot P(S)$? $P(A \cap S) = \frac{25}{75} = .33\bar{3}$

$P(A) = \frac{34}{75}$, $P(S) = \frac{50}{75}$, $P(A) \cdot P(S) = \frac{34}{75} \cdot \frac{50}{75} = \frac{1700}{5625} \approx .302\bar{2}$

do not match, dependent

3. Are the events E and F independent or dependent? Justify your answer with the correct notation and computations. (a) use conditional probabilities; (b) use intersections

(a) Check: Does $P(E|F) = P(E)$? $P(E|F) = \frac{4}{25} \approx .16$,

$P(E) = \frac{14}{75} \approx .18\bar{6}$. They don't match: dependent.

(b) Check: Does $P(E \cap F) = P(E) \cdot P(F)$? $P(E \cap F) = \frac{4}{75} = .05\bar{3}$

$P(E) \cdot P(F) = \frac{14}{75} \cdot \frac{25}{75} = \frac{350}{5625} \approx .062\bar{2}$ don't match, dependent.

SOLUTIONS

4. Now suppose we have a medical test with the following results:

		Positive test	Negative test	total
ill =	Have the disease	37	13	50
healthy =	Do not have the disease	15	285	300
	total	52	298	350

Express each of these as a conditional probability (using the correct probability notation), and then give the answer as a fraction (no need to reduce):

(a) the **sensitivity** (the probability of a positive test, given that the patient has the disease):

$$P(\text{pos test} \mid \text{have disease}) = \frac{37}{50}$$

(b) The **specificity** (the probability of a negative test, given that the patient is well):

$$P(\text{neg test} \mid \text{don't have disease}) = \frac{285}{300}$$

(c) The **PPV** (positive predictive value, the probability they have the disease if they test positive):

$$P(\text{have disease} \mid \text{pos test}) = \frac{37}{52}$$

(d) The **NPV** (Negative predictive value, the probability they are well if they test negative):

$$P(\text{don't have disease} \mid \text{neg test}) = \frac{285}{298}$$

(e) The **false positive rate** (the probability the person tests positive, given that they do not have the disease):

$$P(\text{pos test} \mid \text{don't have disease}) = \frac{15}{300}$$

(f) The **false negative rate** (the probability the person tests negative, given that they have the disease):

$$P(\text{neg test} \mid \text{has disease}) = \frac{13}{50}$$