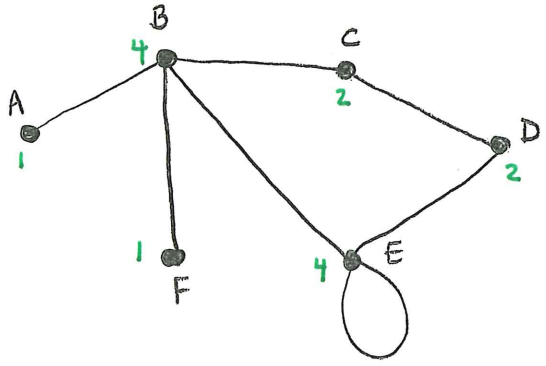
1. Consider the graph shown.




Is the graph simple? NO (has loop)

Is it complete? NO

Is it connected? YES

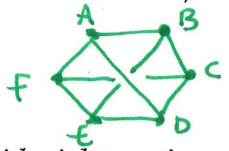
How many components does it have? 1

Does it have an Euler circuit or path? (explain)

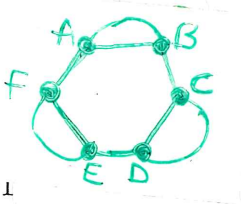
This graph has an Euler path (which will begin/end at A and F) but no Euler circuit. Exactly two vertices have odd degree.

2. Draw a graph which meets the conditions or explain why it cannot be done.

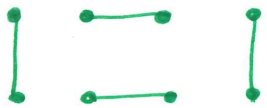
(a) A graph with six vertices, all of which have degree 3




or




(b) A graph with eight vertices, all of which have degree 1



(c) A graph with seven vertices, all of which have degree 1

Not possible: The sum of the degrees is twice the number of edges. This graph would have 3.5 edges.

3. Suppose we have a **complete** graph with 10 vertices.

(a) How many edges does the graph have? (show the formula and the answer)

$$\frac{n(n-1)}{2} = \frac{10(9)}{2} = 45$$

(b) What is the degree of each vertex?

9. (Every vertex is connected to every vertex other than itself.)

(c) Does this graph have an Euler circuit, an Euler path, or neither? (explain)

Neither: it has more than two vertices of odd degree

(d) How many distinct Hamiltonian circuits does it have? (show the formula and the answer)

$$\frac{(n-1)!}{2} = \frac{9!}{2} = \frac{9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2}{2} = 181440$$

SOLUTIONS

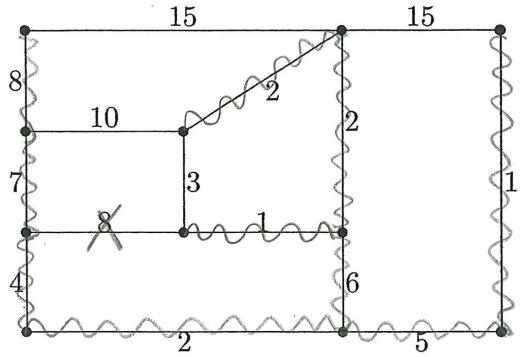
(4) The graph shown has 11 vertices and 15 edges. Use Kruskal's algorithm to find a minimum spanning tree for this graph.

1
1
2
2
2
~~8~~
4
5

closes circuit →

6
7
~~8~~
8 ← STOP with 10 edges
10
15
15

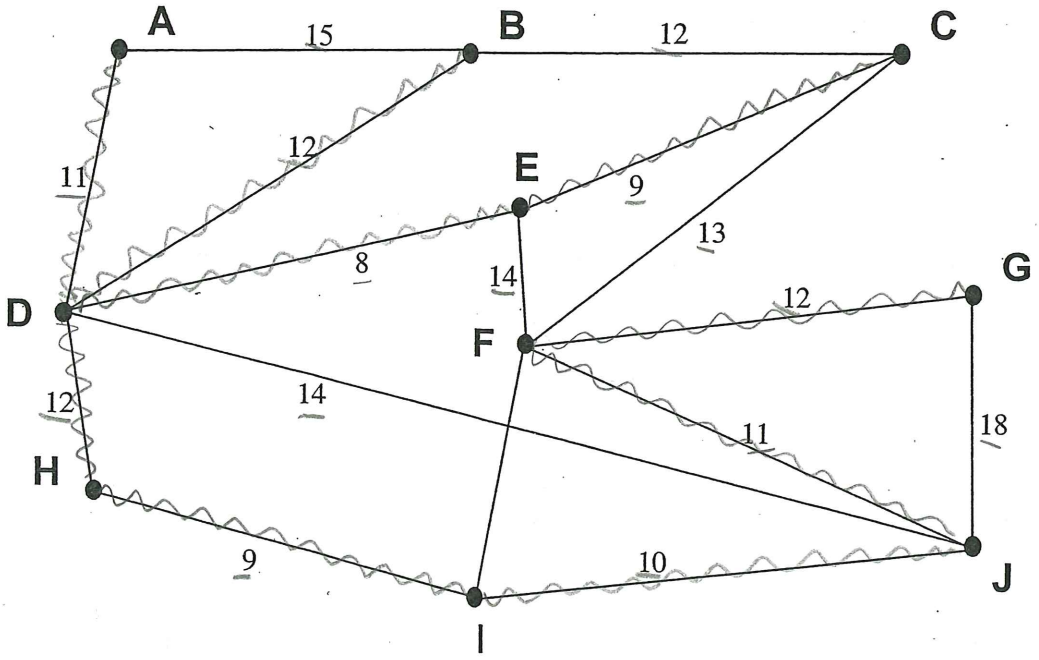
Wavy Lines on graph are solution




(5) (a) How many vertices are in the graph below? 10

(b) How edges will there be in a spanning tree of the graph below? 9

(c) Use Kruskal's algorithm to find a minimum spanning tree for the graph below. List the edges in the order you use them in the algorithm and give the total weight of the minimum spanning tree.




8
9
9
10
11
11
12
12
12
~~12~~
13
14
14
15
18

(Some choices with which of the 12's to use)

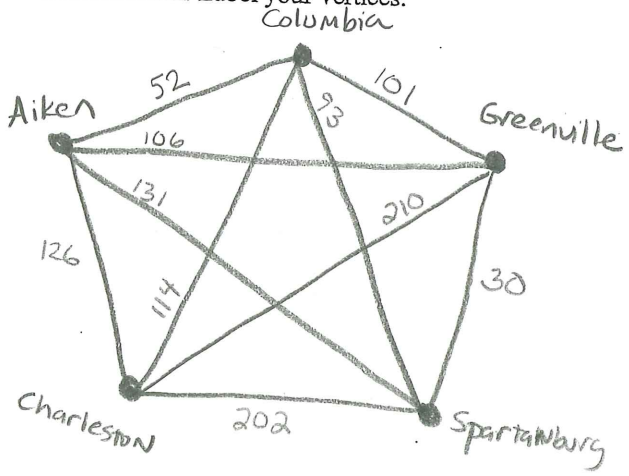
Total weight: 94

SOLUTIONS

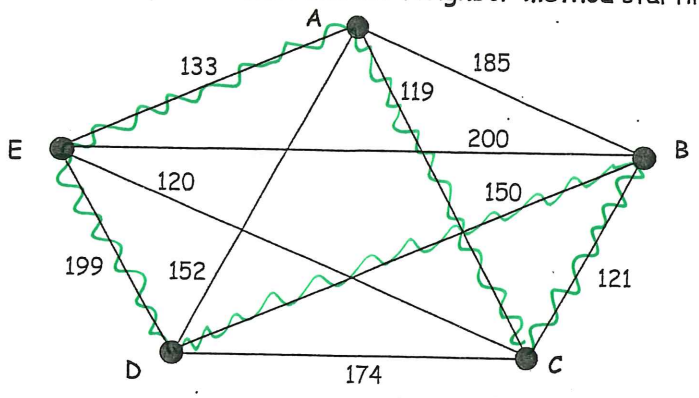
(6) The distances between five cities in South Carolina are given in the table below. The cities are Aiken (A), Columbia (C), Greenville (G), Spartanburg (S), and Charleston (Ch). We will now try to plan the route for a traveling salesman visiting each of these cities.

Draw a weighted complete graph that represents this information. Label your vertices.

| | A | C | G | S | Ch |
|---|---|---|---|---|---|
| A | -- | 52 | 106 | 131 | 126 |
| C | 52 | -- | 101 | 93 | 114 |
| G | 106 | 101 | -- | 30 | 210 |
| S | 131 | 93 | 30 | -- | 202 |
| Ch | 126 | 114 | 210 | 202 | -- |




(7) **Hamiltonian Circuit.** Jim the Shoeshine Sales Representative must display the latest shoeshine products in four distant cities before returning home. He is taking the company jet, and has been ordered to choose the route that will have the minimum travel time (and thus the least fuel). Use the **Nearest Neighbor method** starting at B.



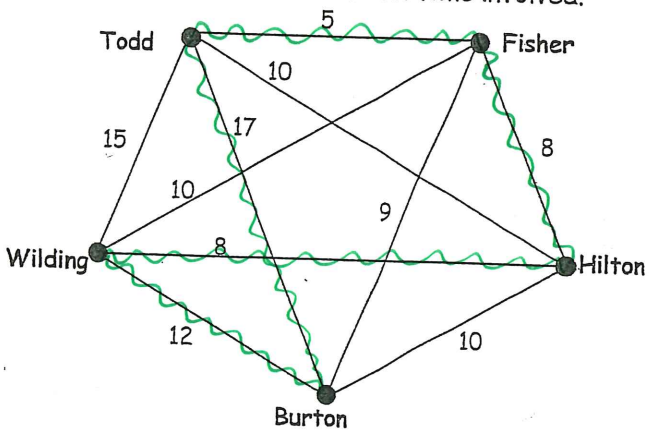

BC 121
CA 119
AE 133
ED 199
DB 150

Total: 722

(8) **Hamiltonian Circuit.** Elizabeth Taylor plans to visit five different grave sites of five different husbands. She demands a route which will minimize the travel time involved.

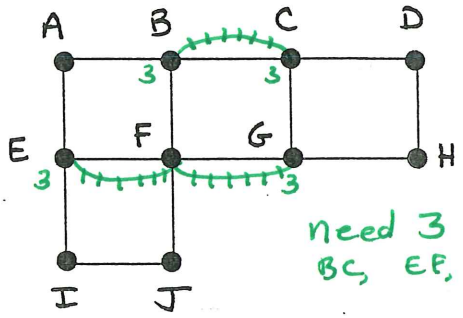
Use the **Cheapest Link** (aka Sorted Edges) algorithm to find a possible circuit.

5
8
8
~~X~~ 3 at F
~~X~~ 3 at H
~~X~~ 3 at H
~~X~~ 3 at F
12
~~X~~ 3 at W
17



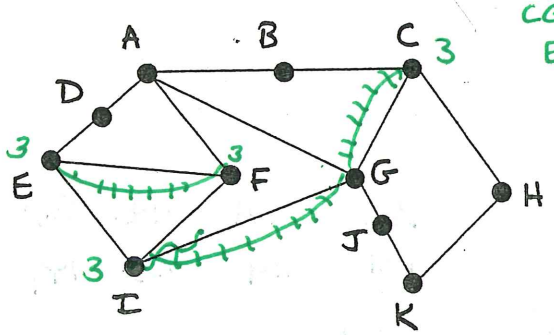
SOLUTIONS

(10) ...d **minimal Eulerizations** for each of the following graphs.

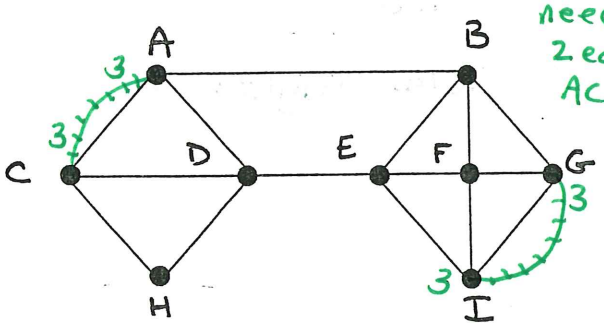



(one possibility)

need 3 edges: BC, EF, FG.



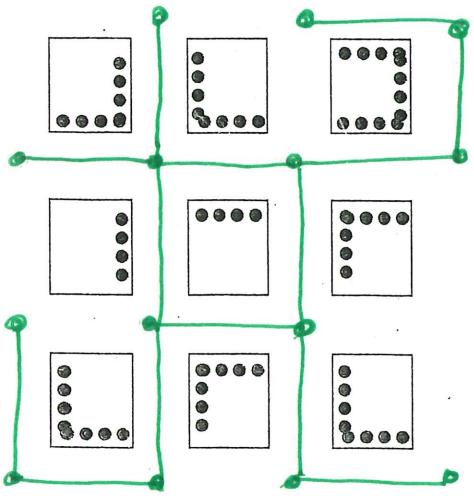

needs 3 edges: CG, GI, EF



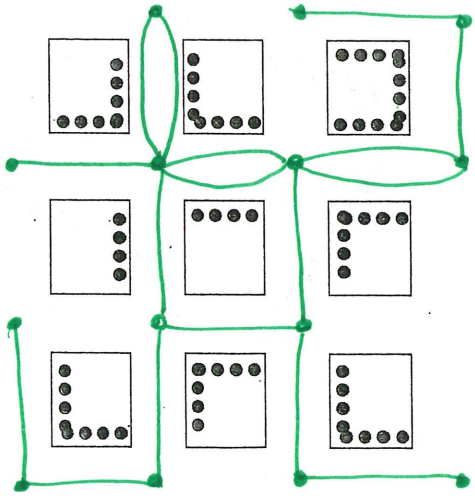

needs 2 edges AC, GI

(11) Find the graph model.

Version 1: "Garbage Truck": we need to pass down each occupied street once



Version 2: "Meter Reader / Mail Carrier": we need to pass down *each occupied side* of each street



(12) **Euler Circuit.** The Night Watchman must choose a route to patrol an office building by walking down every hallway to peer in each of the rooms before returning to the guard station.

If this graph represents the hallways, Eulerize it, and then find an Euler circuit which begins and ends at the guard station.

add 3 edges