

**Yet Another Voting Worksheet: Focus on
Independence of Irrelevant Alternatives Criterion (IIA)**

Please form groups of 2 to 4 people. Turn in one page per group. Blank copies and solutions will be posted on our course website.

Names:

SOLUTIONS

A class must choose a new pet. Their choices are an Aardvark, a Bear, a Camel, a Donkey, or an Elephant. They are asked to rank their choices, and the ballots are found as shown:

1. We decide to use the Borda Count method. Show work to find the Borda points for each pet. Which pet will we choose?

$$A: 8(3) + 6(5) + 5(1) + 5(3) + 2(4) = 82$$

$$B: 8(4) + 6(2) + 5(2) + 5(1) + 2(2) = 63$$

$$C: 8(5) + 6(1) + 5(4) + 5(4) + 2(1) = 88$$

$$D: 8(2) + 6(3) + 5(3) + 5(5) + 2(5) = 84$$

$$E: 8(1) + 6(4) + 5(5) + 5(2) + 2(3) = 73$$

We will choose a CAMEL.

	8	6	5	5	2
1 st	C	A	E	D	D
2 nd	B	E	C	C	A
3 rd	A	D	D	A	E
4 th	D	B	B	E	B
5 th	E	C	A	B	C

2. Does our election using Borda points violate the Majority Criterion? Why or why not?

NO: Because there is no majority candidate, it is impossible to violate the majority criterion.

3. After the election, someone discovers that the store is all out of pet Aardvarks. Write the new preference schedule without the aardvarks, and find the Borda points. Which pet wins now?

8	6	5	5	2
C	E	E	D	D
B	D	C	C	E
D	B	D	E	B
E	C	B	B	C

$$B: 8(3) + 6(2) + 5(1) + 5(1) + 2(2) = 50$$

$$C: 8(4) + 6(1) + 5(3) + 5(3) + 2(1) = 70$$

$$D: 8(2) + 6(3) + 5(2) + 5(4) + 2(4) = 72$$

$$E: 8(1) + 6(4) + 5(4) + 5(2) + 2(3) = 68$$

The donkey wins this election.

4. Explain why this election shows that the Borda Count method can violate the Independence of Irrelevant Alternatives Criterion (IIA).

The aardvark lost the original election. If Borda Count satisfied IIA, then removing the aardvark should not change the winner. Instead, with no one changing their votes, removing the aardvark caused the winner to change from the camel to a donkey.

5. Now suppose they decided, instead, to use the Method of Pairwise Comparisons. Perform the comparisons and list the pairwise points for the original election. (You have a group: divide the work on this!)

Hint: exactly three of the matchups are ties.

	8	6	5	5	2
1 st	C	A	E	D	D
2 nd	B	E	C	C	A
3 rd	A	D	D	A	E
4 th	D	B	B	E	B
5 th	E	C	A	B	C

AARDVARKS IN HERE

TIE A vs B: $\frac{8\ 6\ 5\ 5\ 2}{B\ A\ B\ A\ A}$
A 13, B 13

A vs. C: $\frac{8\ 6\ 5\ 5\ 2}{C\ A\ C\ C\ A}$
C 18, A 8

A vs. D: $\frac{8\ 6\ 5\ 5\ 2}{A\ A\ D\ D\ D}$
A 14, D 12

A vs. E: $\frac{8\ 6\ 5\ 5\ 2}{A\ A\ E\ A\ A}$
A 21, E 5

B vs. C: $\frac{8\ 6\ 5\ 5\ 2}{C\ B\ C\ C\ B}$
B 8, C 18

B vs. D: $\frac{8\ 6\ 5\ 5\ 2}{B\ D\ D\ D\ D}$
B 8, D 18

B vs. E: $\frac{8\ 6\ 5\ 5\ 2}{B\ E\ E\ E\ E}$
B 8, E 18

TIE C vs. D: $\frac{8\ 6\ 5\ 5\ 2}{C\ D\ C\ D\ D}$
C 13, D 13

TIE C vs. E: $\frac{8\ 6\ 5\ 5\ 2}{C\ E\ E\ C\ E}$
C 13, D 13

D vs. E: $\frac{8\ 6\ 5\ 5\ 2}{D\ E\ E\ D\ D}$
D 15, E 11

Pairwise Points (make sure they add to 10):

A: $2\frac{1}{2}$ B: $\frac{1}{2}$ C: 3 D: $2\frac{1}{2}$ E: $1\frac{1}{2}$ Winner? CAMEL

6. Now someone finds out it's actually illegal to own a pet Aardvark (the others, of course, are all perfectly fine). Use the method of pairwise comparisons without the aardvarks. (Hint: notice that you don't need to redo the individual comparisons: you just need to recount the pairwise points.)

Pairwise Points (make sure they add to 6):

B: 0 C: 2 D: $2\frac{1}{2}$ E: $1\frac{1}{2}$ Winner? DONKEY

7. For the original election, C wins versus plurality with 8 1st place votes. If we eliminate D, C will still win with 13 first place votes. Does this mean that plurality method satisfies IIA? Explain.

No. While this particular scenario doesn't change the winner, it is possible that a different elimination (or different election) might not. We only say a voting method satisfies a fairness criterion like IIA if it holds for every possible election.