

Review of the Four Fairness Criteria

Majority Criterion: If candidate X has a majority of the first-place votes, then candidate X should be the winner of the election.

- The majority criterion is always satisfied by the Plurality Method, the Plurality with Elimination Method, and Pairwise Comparison Method.
- The Borda Count Method does not satisfy the majority criterion. This means that the Borda Count Method does not always select the candidate with the majority of first place rankings.

Condorcet Criterion: If candidate X is preferred by the voters over each of the other candidates in a head-to-head comparison, then candidate X should be the winner of the election.

- The Condorcet criterion is always satisfied by the Method of Pairwise Comparison.
- The Borda Count Method, the Plurality with Elimination Method, and the Plurality Method might select a Condorcet candidate, but they can also fail to honor the criterion.

Monotonicity Criterion: If candidate X is a winner of an election and, in a reelection, the only changes in the ballots are changes that favor X (and only X), then X should still be the winner.

- The Plurality Method, the Borda Count Method and the Pairwise Comparison Method always satisfy the monotonicity criterion.
- The Plurality with Elimination Method can violate the monotonicity criterion.

Independence of Irrelevant Alternatives Criterion (IIA): If candidate X is a winner of an election and in a recount one of the nonwinning candidates withdraws or is disqualified, then X should still be the winner.

- The Plurality Method, the Borda Count Method, the Pairwise Comparison Method, and the Plurality with Elimination Method can fail to satisfy the Irrelevant Alternative criterion.

Note: the following table should help organize your thoughts. For each example of violation, see if you can construct an example. You should understand this table and what it means, but you do not need to memorize it.

	Majority	Condorcet	Monotonicity	Ind. of Irr. Alt.
Plurality	satisfy	violate	satisfy	violate
Borda	violate	violate	satisfy	violate
Plur. w/ Elim.	satisfy	violate	violate	violate
Pairwise comp.	satisfy	satisfy	satisfy	violate

Arrow's Impossibility Theorem: It is mathematically impossible for a democratic voting method to satisfy all four of the fairness criteria.

- Maybe his conditions are too strict? Since Arrow's original work, many investigators have developed a wide array of desirable fairness rules and showed results similar to Arrow's theorem.
- In 1972 Arrow was awarded the Nobel Prize in Economics (there is no Nobel Prize in Mathematics) for his pioneering work in what is now known as social-choice theory, a discipline that combines aspects of mathematics, economics, and political science.