SOLUTIONS

Names:

Joe, Moe and Bo inherited a spitting camel and a hand-woven tapestry from their Uncle Zanzibar, with the stipulation that they must divide these items fairly, without either item leaving the family. The table below shows what each of these is worth them.

	Joe	Moe	Bo
camel	$3000	$2400	$1700
tapestry	$600	$1000	$900

1. Find the total bids b_{Joe}, b_{Moe}, b_{Bo} and also compute what each of them thinks is a **fair share**.

   $b_{Joe} = 3600$
   $b_{Moe} = 3400$
   $b_{Bo} = 2600$

   Fair Shares: Joe $1200
   (divide total   Moe $1133.33
   bid by 3)   Bo $866.67

2. Consider the following settlement: Joe gets the camel and pays $1200, Moe receives $1200, Bo gets the tapestry (and doesn't pay or receive any cash). **Is this settlement fair?** Explain and show work.

   x_{Joe} = $\underset{camel}{3000} - \underset{cash}{\underset{payment}{1200}}$ = $1800, more than $1200

   x_{Moe} = 1200, more than $1133.33

   x_{Bo} = $\underset{tapestry}{900}$, more than $866.67

   This settlement is fair since all three have at least their fair share

3. Suppose the brothers decide to use **Knaster's method of sealed bids**.

   a. Find the **first settlement**: which goods will each person receive, and how much cash will they pay or receive?

      Joe gets the camel and pays $\underset{camel}{3000} - \underset{fairshare}{1200} = \underline{\$1800}$

      Moe gets the tapestry and <u>receives</u> $133.33

      Bo gets no items, <u>receives</u> $866.67

   b. Find the **surplus**, the total paid – total received.

      $1800 - (133.33 + 866.67) = \boxed{\$800}$

   c. Find the final settlement (divide the surplus equally among the three players.) Give one sentence for each person: which items they have, and the total cash they pay or receive.

      $\left(\dfrac{800}{3} = 266.67\right)$ Joe gets the camel and <u>pays</u> $1533.33 ← take 1st settlement 1800, subtract 266.67

      Moe gets the tapestry and <u>receives</u> $400

      Bo <u>receives</u> $1133.34

4. From your final settlement above, find the **net value** of their perceived winnings (value of items plus/minus cash receive/paid), also called their **compensation**: x_{Joe}, x_{Moe} and x_{Bo}.

$$x_{Joe} = 3000 - 1533.34 = \$1466.67$$

$$x_{Moe} = 1000 + 400 = \$1400$$

$$x_{Bo} = \$1133.34$$

5. Find the **XB ratios** for Joe, Moe and Bo. (For each player, compute x_{Player}/b_{Player}.)

Joe: $\dfrac{1466.67}{3600} = .4074$ Bo: $\dfrac{1133.34}{2600} = .4359$

Moe: $\dfrac{1400}{3400} = .4118$ (all are over $\frac{1}{3} = .33\overline{3}$, so settlement is fair)

6. Next use **the Equitability method** to divide the surplus you found in problem 3b instead of Knaster's method. **How much of the surplus** will each player receive? (First, find the total of all the players' bids.)   Total of all bids: 9600

Joe's share: $\dfrac{3600}{9600} \cdot 800 = \$300$

Moe's share: $\dfrac{3400}{9600} \cdot 800 = \$283.33$ } check: these add to $800

Bo's share: $\dfrac{2600}{9600} \cdot 800 = \$216.67$

7. Give the final settlement using the **Equitability Method**: which goods each person receives, along with the net cash they pay or receive. (Begin with your settlement in 3a, and add the surplus amounts you found in problem 6 above.)

Joe gets the camel, pays $1800 - 300 = \underline{1500}$

Moe gets the tapestry, receives $133.33 + 283.33 = \$\underline{416.67}$

Bo gets $866.67 + 216.67 = \$\underline{1083.34}$

(Check: total paid (1500) matches total received: $1083.34 + 416.67$.)

8. Find x_{Joe}, x_{Moe}, x_{Bo} and the XB ratios for each, based on the settlement you found in problem 7.

$$x_{Joe} = 3000 - 1500 = \$1500$$

$$x_{Moe} = 1000 + 416.67 = \$1416.67$$

$$x_{Bo} = \$1083.34$$

Joe's XB: $\dfrac{1500}{3600} = .4167$

Moe's XB: $\dfrac{1416.67}{3400} = .4167$

Bo's XB: $\dfrac{1083.34}{2600} = .4167$

(The settlement is equitable: all match!)