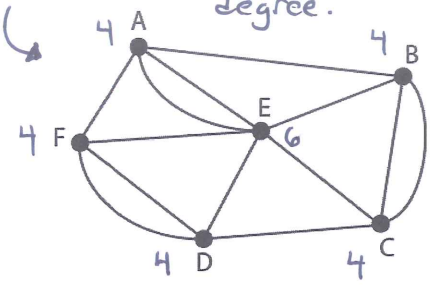
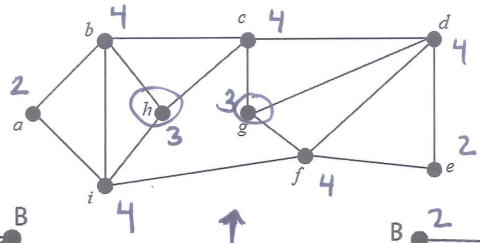
Math 111 Second Graph Theory Worksheet

Names: SOLUTIONS

I. Which of the following three graphs have Euler circuits? Which have an Euler path but no circuit? Which have neither? Briefly justify, but you **do not** need to actually find the circuit or path.

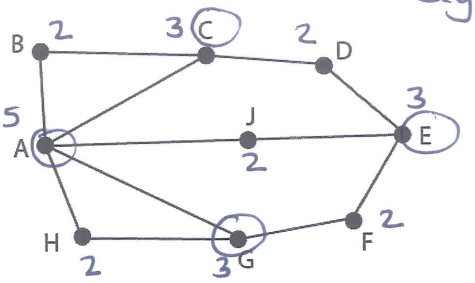
Has Euler circuit: all vertices have even degree.





Has Euler path but no circuit: exactly two vertices with odd degree

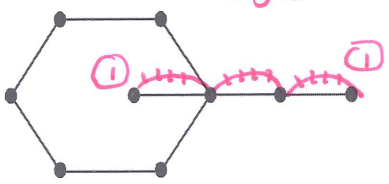
Has Neither Euler path NOR circuit: more than 2 vertices with odd degree



II. For each of the following graphs, Eulerize by adding only **legal** edges (i.e., duplicating existing edges). Try to find an optimal solution (duplicate the fewest number of edges). *Hint*: first label the degree of each vertex.

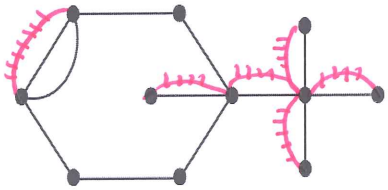
Graph A

add 3 edges



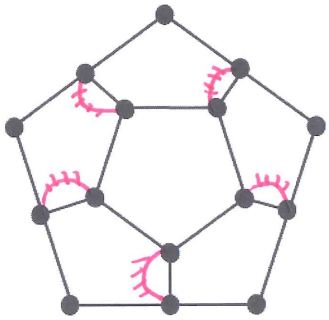
Graph B

add 6 edges



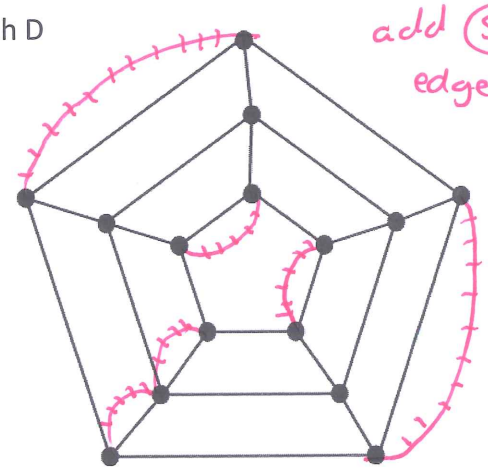
Graph C

add 5 edges



Graph D

add six edges

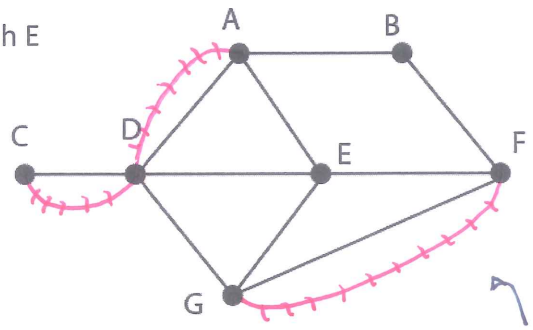


this is one option; there are other ways!

SOLUTIONS

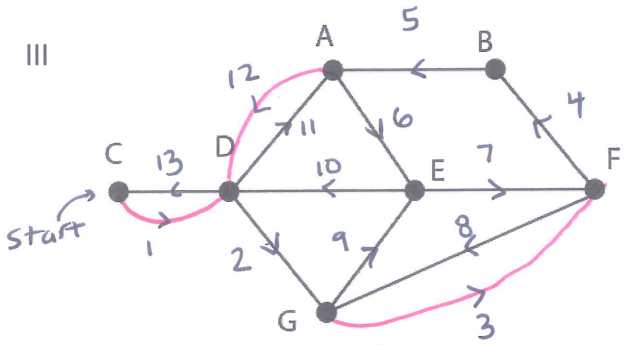
Math 111 Second Graph Theory Worksheet *the back*

Graph E



Add 3 edges (other solutions possible!)

III



III. For graph E above, **find an Euler circuit on your new graph**. Add the necessary edge(s), and then **label your circuit** by numbering the edges and drawing arrows to indicate direction.

IV. Suppose a graph has vertices with degrees 2, 3, 3, 3, 5. How many edges does the graph have?

sum is 16

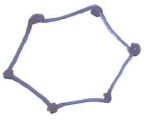
sum of degrees = 2 · (# edges) ⇒ 16 = 2e

$\boxed{e = 8}$

V. For each of these, **construct** a graph with the required properties.

a. Graph has six vertices, all of degree 2

one option:



another option:

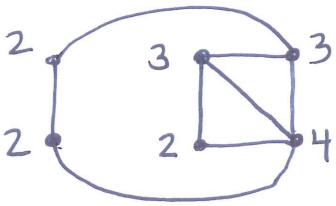
a third choice:

b. Graph has six vertices, all of degree 1

only option:

c. Graph has six vertices, is connected, with degrees 2, 2, 2, 3, 3, 4.

one option:



must have exactly two odd degrees

d. Graph has seven vertices, is connected, and has an Euler path but no Euler circuit.

one option: