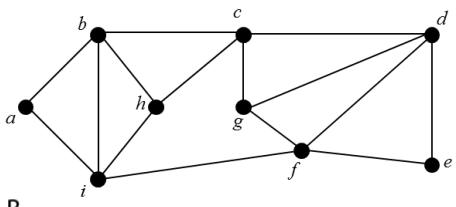
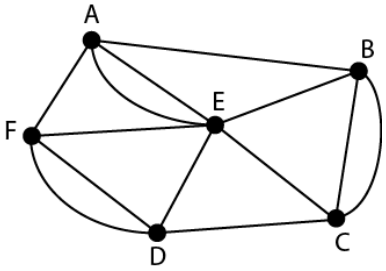
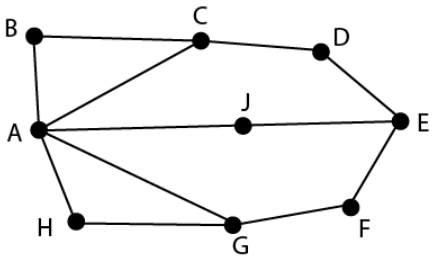
Math 111 Second Graph Theory Worksheet

Names:

I. Which of the following three graphs have Euler circuits? Which have an Euler path but no circuit? Which have neither? Briefly justify, but you **do not** need to actually find the circuit or path.

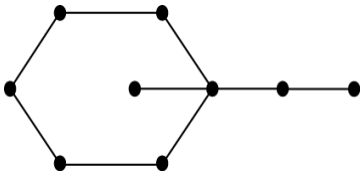




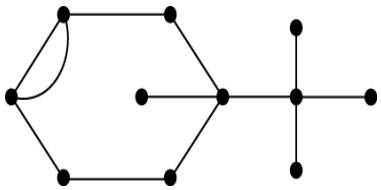


II. For each of the following graphs, Eulerize by adding only **legal** edges (i.e., duplicating existing edges). Try to find an optimal solution (duplicate the fewest number of edges). *Hint*: first label the degree of each vertex.

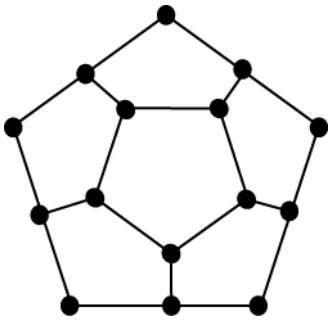
Graph A



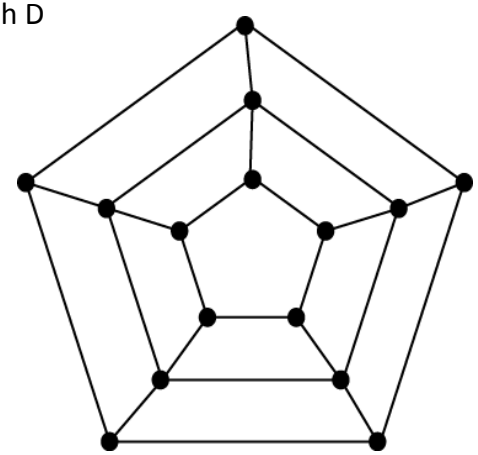
Graph B



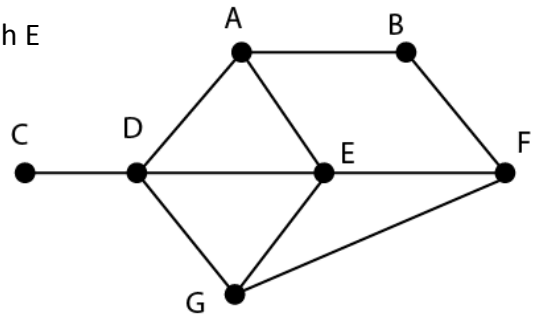
Graph C



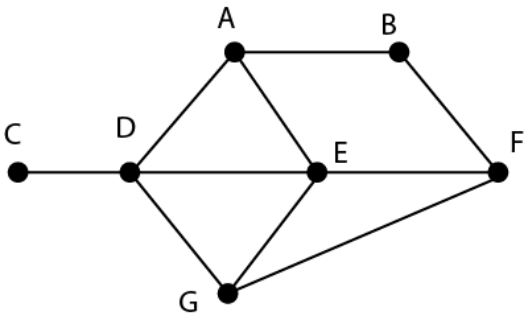
Graph D



Graph E



III



III. For graph E above, **find an Euler circuit on your new graph**. Add the necessary edge(s), and then **label your circuit** by numbering the edges and drawing arrows to indicate direction.

IV. Suppose a graph has vertices with degrees 2, 3, 3, 3, 5. How many edges does the graph have?

V. For each of these, **construct** a graph with the required properties.

a. Graph has six vertices, all of degree 2

b. Graph has six vertices, all of degree 1

c. Graph has six vertices, is connected, with degrees 2, 2, 2, 3, 3, 4.

d. Graph has seven vertices, is connected, and has an Euler path but no Euler circuit.