Math 111 – 013 September 16, 2015

Names: SOLUTIONS

An election has four candidates (A, B, C and D).

	7	7	8	5
1st	A	C	D	B
2nd	B	B	C	A
3rd	C	A	B	C
4th	D	D	A	D

Winner using each method:
Plurality: D
Plurality with Elimination: ? ~~A~~
Borda: B
Pairwise Comparisons: C

1. Is there a majority candidate?

Total votes: 27 # needed for majority: 14

Actual 1st place votes: A 7, B 5, C 7 D 8.

NO, there is no majority candidate.

2. Find the winner using Plurality with Elimination.

No majority, so eliminate B:

7	7	8	5
A	C	D	A
C	A	C	C
D	D	A	D

1st place votes: A 12, C 7, D 8.

still no majority, so eliminate C:

7	7	8	5
A	A	D	A
D	D	A	D

A: 7+7+5 = 19

D 8

Winner is A

3. Is there a Condorcet Candidate? (Since C wins using pairwise, he is the only possibility. Check all pairs with candidate C to be sure.)

C 7+8=15 vs A 7+5=12

C 7+8=15 vs B 7+5=12

C 7+7+5=19 vs D 8

Yes: C wins all head-to-head matchups, so he is a Condorcet candidate.

4. Suppose we declare D to be the winner. Have we violated the Condorcet Criterion?

Yes. We have a Condorcet candidate C, so if we let D win we have violated this criterion.

5. Suppose instead we declared B to be the winner. Have we violated the majority criterion? Explain.

No. There is no majority candidate, so it is impossible to violate the majority criterion.

Note: Based on the results in the box above, this election shows that Plurality, Plurality w/ Elimination, and Borda methods all can violate the Condorcet criterion.