Print **all** group member's names here. Circle the name of the group member who turns this in.

SOLUTIONS

1. A game costs \$3 to play: we roll a standard, fair 6-sided die. If we get an even number, we win \$2. If we roll a 5, we win \$9. Otherwise, we win nothing. What is the **expected value** for this game? **Show all steps**. Over the long term, would you rather be the player, or the person running the game? Possible outcomes: \$2, \$9, or \$0

expected winnings: $\$2 \cdot \frac{3}{6} + \$9 \cdot \frac{1}{6} + \$0 \cdot \frac{2}{6}$

$= \frac{6}{6} + \frac{9}{6} + 0 = \frac{15}{6} = \$2.50$

expected value $= 2.50 - 3.00 = \boxed{\$ -.50}$ ← over the long term, player will lose on average 50¢ per game.

cost

We would rather run the game – they'll gain money long term.

2. Consider the following table. Find each probability, and express with appropriate notation.

education level	Less than high school graduate (L)	High school graduate (H)	Some college	College graduate (G)	*total*
Smartphone (S)	5	20	12	22	59
Cellphone, not smartphone (C)	4	7	3	2	16
No phone (N)	1	3	0	1	5
total	10	30	15	25	80

Suppose we choose a person at random.

(a) What is the probability they do not own a phone, or that they are a high school graduate?

$\boxed{P(N \cup H)} = P(N) + P(H) - P(N \cap H) = \frac{5}{80} + \frac{30}{80} - \frac{3}{80} = \boxed{\frac{32}{80}}$

(b) What is the probability they own a smartphone, given that we know their education level is "some college"?

$\boxed{P(S \mid \text{Some College}) = \frac{12}{15}}$

(c) Suppose one of the high school graduates who does not own a phone were to leave on vacation. We pick one of the remaining people at random. What is the probability they are a college graduate?

$P\left(\begin{matrix}\text{2nd person} \\ \text{college grad}\end{matrix} \middle| \begin{matrix}\text{1st person} \\ H \cap N\end{matrix}\right) = \boxed{\frac{25}{79}}$

Suppose we choose two different people at random (without replacement).

(d) What is the probability that neither person owns a phone?

$P\left(\begin{matrix}\text{1st person} \\ N\end{matrix} \cap \begin{matrix}\text{2nd person} \\ N\end{matrix}\right) = \frac{5}{80} \cdot \frac{4}{79} = \boxed{\frac{20}{6320}} = \frac{1}{316}$

(a) What is the probability that the first person is a college graduate, and the second person is a high school graduate?

$P\left(\begin{matrix}\text{1st person} \\ G\end{matrix} \cap \begin{matrix}\text{2nd person} \\ H\end{matrix}\right) = \frac{25}{80} \cdot \frac{30}{79} = \boxed{\frac{750}{6320}}$

SOLUTIONS

3. A deck of cards has 4 suits, A, B, C, and D. Suits A and B have cards 1—8. Suit C has cards 1, 3, 5, 7, and 9; suit D has cards 2, 4, and 6 and 8.

(a) Suppose we draw a card at random. What is the probability we draw an **even** number? What is the probability we draw an **odd** number?

\# of cards: $\underset{A}{8} + \underset{B}{8} + \underset{C}{5} + \underset{D}{4} = 25$ cards in the deck

$P(\text{even}) = \frac{4+4+0+4}{25} = \boxed{\frac{12}{25}}$ $P(\text{odd}) = \frac{4+4+5+0}{25} = \boxed{\frac{13}{25}}$

(b) We play a game, which costs \$6 to play. If you draw an odd number you win \$5, and if you draw an even number you get \$10. What is the **expected value** for this game?

expected winnings: $\$5\left(\frac{13}{25}\right) + \$10\left(\frac{12}{25}\right)$

$= \frac{65+120}{25} = \frac{185}{25} = \$7.40$ expected value $= 7.40 - 6$ $\boxed{= \$1.40}$

(c) Suppose we draw one card at random. Let B be the event that the card we draw is suit B; let "4" be the event that the card is a 4. Find the following:

$P(B \cup \text{"4"}) = \frac{8}{25} + \frac{3}{25} - \frac{1}{25} = \boxed{\frac{10}{25}}$ $P(B \cap \text{"4"}) = \boxed{\frac{1}{25}}$

$P(B | \text{"4"}) = \boxed{\frac{1}{3}}$ $P(\text{"4"} | B) = \boxed{\frac{1}{8}}$

Are B and "4" **independent** events? Explain clearly and do any needed calculations.

$P(B) = 8/25$ $P(B|4) = \frac{1}{3}$ These are not exactly equal: the events are <u>Not</u> independent.

(or, compare $P(4) = 3/25$ and $P(4|B) = \frac{1}{8}$: also not exactly equal.)

$P(C | \text{"9"}) = \frac{1}{1} = 1$ $P(\text{"9"} | C) = \boxed{\frac{1}{5}}$

(d) Suppose we draw two cards in a row without replacement. What is the probability the first card is suit A and the second card is suit D?

$P(1^{st} \text{ is A} \cap 2^{nd} \text{ is D}) = \frac{8}{25} \cdot \frac{4}{24} = \boxed{\frac{32}{600}} = \frac{4}{75}$

(e) Suppose we draw two cards in a row without replacement. What is the probability that both cards are suit D?

$P(1^{st} \text{ is D} \cap 2^{nd} \text{ is D}) = \frac{4}{25} \cdot \frac{3}{24} = \boxed{\frac{12}{600}} = \frac{1}{50}$

SOLUTIONS

4. For this question, refer to the Fourth Probability Worksheet to find verbal descriptions of each of the medical probability terms.

A group of 500 patients participate in an experiment involving a screening test for a disease. Using the test, 430 people tested positive. We know that the Positive Predictive Value (PPV) is 80%, and the Negative Predictive Value (NPV) is 70%. $\frac{y}{70} = \frac{70}{100}$ $y = .7(70) = 49$ $\frac{x}{430} = \frac{80}{100}$ $x = .8(430) = 344$

(a) Recreate the data from the trial. When needed, round to the nearest whole number.

	Positive test	Negative test	*total*
Have the disease	$x = 344$	21	365
Do not have the disease	86	$y = 49$	135
total	430	70	500

(b) Use the filled-in table above to find the following. **Express each of these as a conditional probability**, and give the answer as a fraction (no need to reduce):

the **sensitivity:**

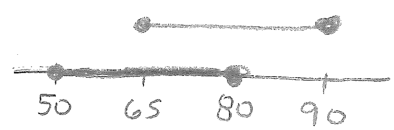
$P(\text{pos test} \mid \text{has disease}) = \boxed{\frac{344}{365}}$

False positive rate:

$P(\text{pos test} \mid \text{doesn't have disease}) = \boxed{\frac{86}{135}}$

5. We are choosing a number at random from the interval $[25,100]$. Assume every real number in the interval is equally likely to be chosen. For each problem below, draw the appropriate number line, simplify the interval if possible, and give the probability as a fraction (no need to reduce). What is the **probability** we select a number in the following intervals?

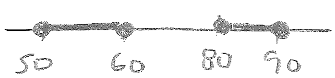
a. $[50,80]\cap[65,90]$



$[65,80]$ $P = \frac{80-65}{100-25} = \boxed{\frac{15}{75}}$

b. $[50,60]\cup[80,90]$

doesn't simplify $P = \frac{(60-50)+(90-80)}{75} = \boxed{\frac{20}{75}}$



c. $[20,60]$, given that it is in $[30,90]$

intersection is $[30,60]$ $P = \frac{60-30}{90-30} = \boxed{\frac{30}{60}} = \frac{1}{2}$