

Names:

SOLUTIONS



Last time we helped Brian, Stewart and Peter divide some office equipment using Knaster's method.

Recall that for the **final settlement** using **Knaster's method**, Brian received the printer and \$55 cash. Stewart received the copier and paid \$75. Peter received the cabinet and monkey and \$20 cash.

	Brian	Stewart	Peter
cabinet	\$60	\$65	\$80
evil monkey	\$35	\$30	\$40
printer	\$90	\$70	\$50
copier	\$200	\$220	\$200

1. For the final settlement using Knaster's above, find the net value of the winnings for each person (i.e., find x_{Brian} , x_{Stewart} and x_{Peter} .) Then find the XB ratio for each person.

$$x_{\text{Brian}} = \underset{\text{printer}}{90} + \underset{\text{cash}}{55} = \boxed{\$145}$$

$$\frac{x_{\text{Brian}}}{b_{\text{Brian}}} = \frac{145}{385} \approx .3766$$

$$x_{\text{Stewart}} = \underset{\text{copier}}{220} - \underset{\text{paid}}{75} = \boxed{\$145}$$

$$\frac{x_{\text{Stewart}}}{b_{\text{Stewart}}} = \frac{145}{385} \approx .3766$$

$$x_{\text{Peter}} = \underset{\text{cabinet}}{80} + \underset{\text{monkey}}{40} + \underset{\text{cash}}{20} = \boxed{\$140}$$

$$\frac{x_{\text{Peter}}}{b_{\text{Peter}}} = \frac{140}{370} \approx .3784$$

2. The x -values above tell us what Brian's settlement is worth to Brian, and what Peter's settlement is worth to Peter, etc.. Now we'll find perceived values from other points of view.



- a) What is Brian's settlement worth to Peter? Does Peter envy Brian? (Compare your calculation of Peter's opinion of Brian's settlement to x_{Peter} you found above. Which would Peter rather have?)

Brian's settlement: printer + \$55

Peter's opinion: $50 + 55 = \$105$

Peter does not envy Brian, since $x_{\text{Peter}} = \$140$ is more than $\$105$.

- b) What is Peter's settlement worth to Brian? Does Brian envy Peter?

Peter's settlement: get cabinet, monkey, and \$20

Brian's opinion: $60 + 35 + 20 = 115$

Brian does not envy Peter because $x_{\text{Brian}} = \$145$ is more than $\$115$.

- c) What is Stewart's settlement worth to Peter? Does Peter envy Stewart?

Stewart's settlement: get copier, pay \$75

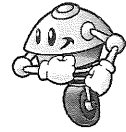
Peter's opinion: $200 - 75 = \$125$

Peter does not envy Stewart because $x_{\text{Peter}} = \$140$ is more than $\$125$.

SOLUTIONS

3. Allie Brosh, Bill Amend, Randall Monroe and Matthew Inman have collaborated to build a giant hamster robot, and now must decide who will get to keep it. Their bids are as follows:

	Allie	Bill	Randall	Matthew
robot	200	600	80	500



Fair shares:

50 150 20 125

a) Who could possibly take the robot and allow for a FAIR settlement? (explain)

average bid = $\frac{200 + 600 + 80 + 500}{4} = 1345$. Bill or Matthew have bids at least this high.

b) Who could possibly take the robot if we also require an ENVY-FREE settlement?

Only Bill (must go to the highest bidder).

c) For an envy-free settlement, how much will each person receive or pay in cash? Give an interval that lists the possible answers.

2nd highest fair share → 125 ≤ possible payment ≤ 150 ✓ highest fair share

Bill receives robot, pays each of the others an amount between \$125 and \$150.

4. Jake, Alisha, and Isaac bought a car during college and shared it for four years. But now they'll move to separate states, and agree that one should keep the car while the others should pay for it. Jake named the car and likes it a lot; to him it is worth \$3000. Alisha likes the car okay; she looked up the trade-in value and thinks it is worth \$2400. Isaac is moving to a far-away state that requires emissions testing, which this car won't pass; it would be expensive to move it and repair it, so to him it is worth only \$1600.



Fair shares: Jake \$1000, Alisha \$800, Isaac \$533.33

a) Find a fair settlement in which Alisha keeps the car. Justify that it is fair to all of them.

one answer:

Alisha gets car and pays Jake \$1000 and Isaac \$550.

$x_{Alisha} = 2400 - 1550 = \850 , more than her fair share. ↑ any amount over \$533.33
 ↑ must compute this

b) For an envy-free settlement, how much will each person receive or pay in cash? Give an interval that lists the possible answers. Then give a particular example of an envy-free settlement.

Envy free: Jake must take car.

2nd highest fair share → 800 ≤ possible payment ≤ 1000 highest fair share

one answer: Jake gets car, pays Alisha and Isaac each \$900.

↑ pick any # in the interval.