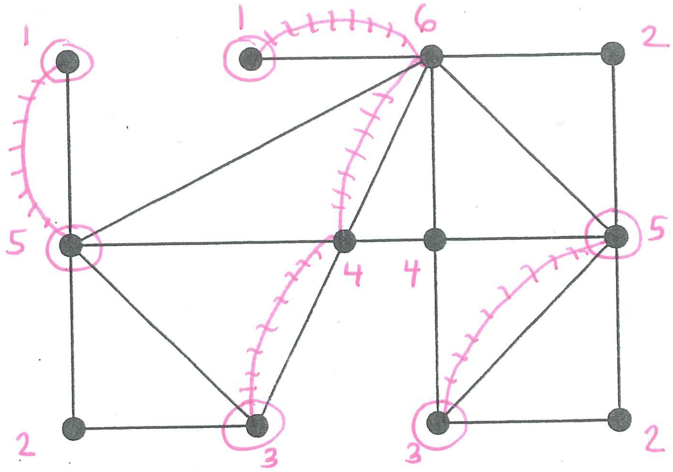
Math 111 Third Graph Theory Worksheet Name: SOLUTIONS

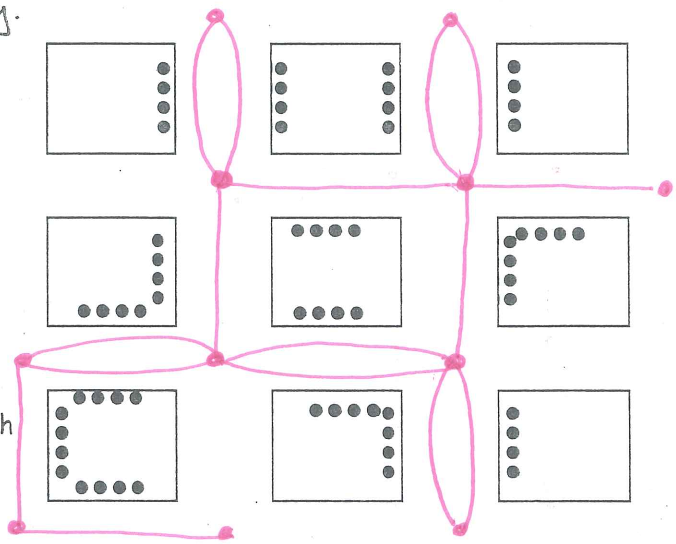
① A Who family wants to visit the Christmas light displays in a neighborhood of Whoville. Their car blows smoke when in reverse, so they don't want to retrace their steps any more than necessary. Find an optimal Eulerization of the graph. Do not find an Euler path or circuit.



(one possibility)

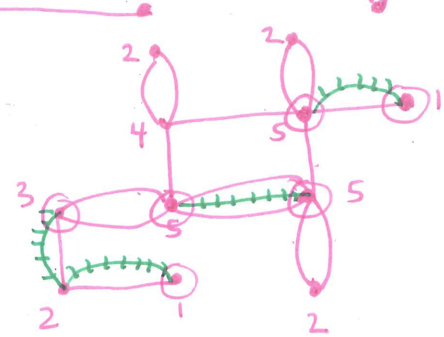
How many edges need to be repeated? 5

② **Euler Circuit.** Billy's newspaper route involves some streets with houses on only one side, and some streets with houses on both sides. He must travel on each side of the street with subscribers separately. Assume that in the diagram at right, the dots represent newspaper subscribers, and the space between the squares represent streets. Also assume that Billy can't travel on the streets with no subscribers - they are part of another boy's route, and he could get in trouble for poaching.



(a) Draw the graph which represents Billy's paper route.

(b) Eulerize this graph: First redraw the graph; then repeat edges as needed. Do Not Find an Euler circuit



← 4 green edges added for Eulerization

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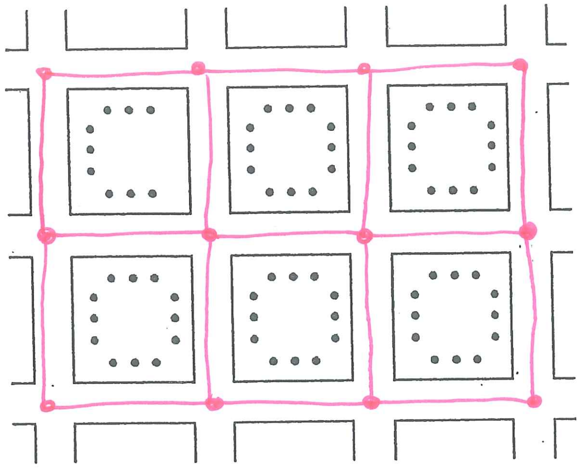
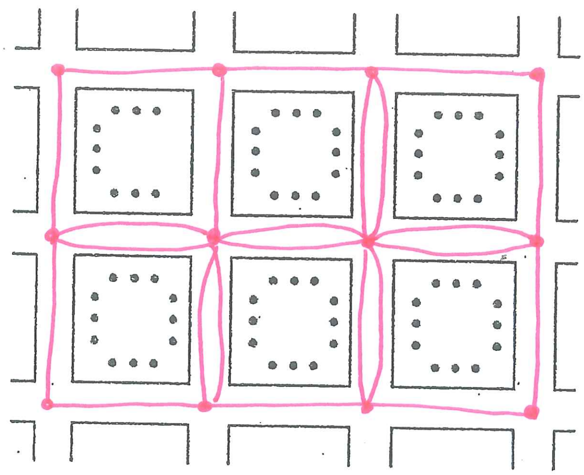
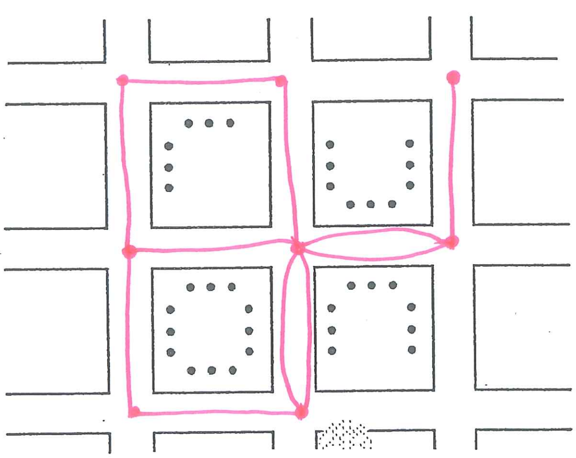
(3) (a) For the street network, draw a graph that would be useful for routing a garbage truck. Assume that all streets are two-way and that passing once down a street suffices to collect from both sides.



(3) (b) Do the same problem on the assumption that one pass down the street only suffices to collect from one side.

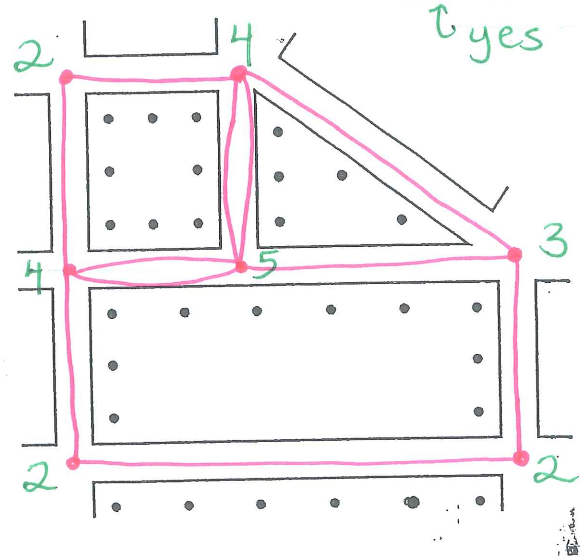


(4) For the street network below, draw the graph that would be useful for finding an efficient route for checking parking meters. (*Hint:* Notice that not every sidewalk has a meter



(5) Draw the graph for the parking-control territory shown in the figure below. Label each vertex with its degree ~~valence~~ and determine if the graph is connected.

yes



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6 1. Consider this graph.

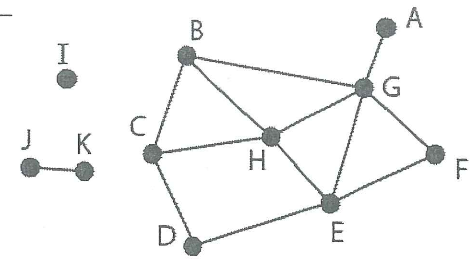
(a) How many **components** does this graph have? 3

(b) Is this graph connected? Circle YES or NO

(c) Is this graph simple? Circle YES or NO

(d) Is this graph a tree? Circle YES or NO

(a tree is connected, and has no circuits)

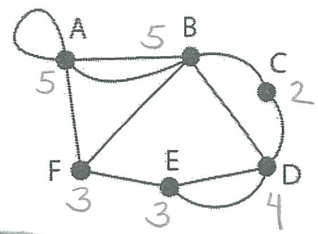


7 2. Consider this graph:

(a) Is this graph simple? Circle YES or NO

(b) Is this graph connected? Circle YES or NO

(c) Find the **degree** of each vertex (label on the graph)



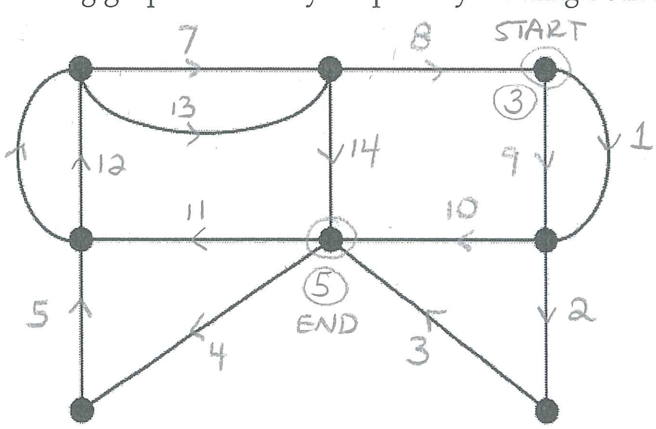
(d) Does this graph have an **Euler Circuit**, an **Euler path**, or **neither**? (circle one)
Explain how you know this without actually finding a circuit or path.
Hint: look at the degree of the vertices you found in part (c).

More than two vertices have odd degree.

8 3. Find an **Euler path** for the following graph. Indicate your path by labeling both arrows and numbers on the edges.

one example:

(The circled odd-degree vertices must be the start and end.)



9 4. A graph has vertices with degrees 1, 1, 2, 2, 3, 5. How many edges does it have?
Hint: use the **formula** that relates the degrees to the number of edges. Do not try to draw the graph.

Sum of degrees: $1+1+2+2+3+5 = 14$

= Twice the # edges, so $2e = 14 \Rightarrow e = 7$

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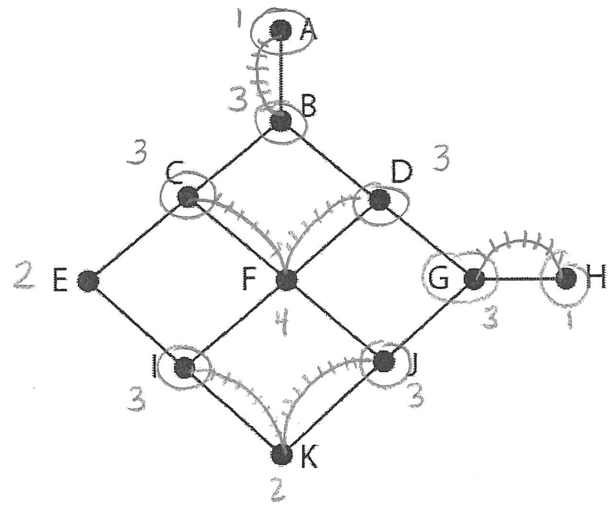
10 5. Eulerize the following graph by adding **legal** edges. Try to find an optimal Eulerization (duplicate the fewest possible edges). **DO NOT** actually find an Euler circuit.

How many edges did you add? 6

List the edges that you added:
(For example, if you added an edge between vertices A and B, you'd write AB.)

One example:

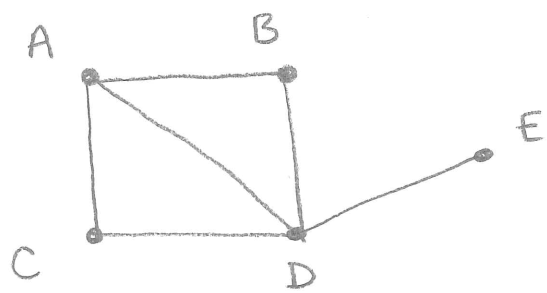
AB, CF, DF, GH, IK, JK



11 6. Construct a **simple, connected** graph that has vertices A, B, C, D, E with degrees 3, 2, 2, 4, 1 **respectively** (i.e., degA = 3, degB = 2, etc.). Clearly label your vertices.

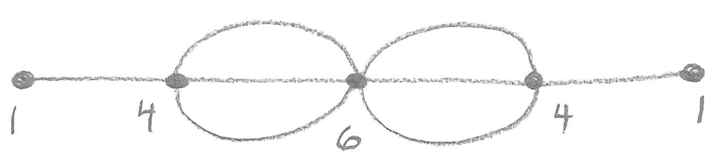
A B C D E

only possibility:



12 7. Construct a connected graph with 5 vertices and at least 8 edges that has an Euler path but no Euler circuit. Clearly label the degree of each vertex.

One possibility:



must have exactly two odd-degree vertices

13. Tree, six vertices, two with degree 3: (connected, with nothing enclosed)