Math 111 Constructing Fairness Examples September 6, 2016

Names: POSSIBLE EXAMPLES

Today we construct more examples, especially to illustrate certain ideas of fairness. For each scenario below, suppose an election has four candidates: Christopher, David, Matt and Peter. **Construct an example** preference schedule **and choose one of our four voting methods** so that the statement comes true. Your preference schedule should have between 20 and 30 voters, and you should **show steps to justify that the statement is true**. There are many possible correct answers to these.

1. David gets the most first place votes, but David doesn't win the election.

   Preference Schedule:

   this example was done in class!
   See your class notes.

   Who has the most first place votes: Christopher, Matt or Peter?
   Which voting method(s) ~~have~~ don't have David as the winner? (show work to confirm David wins.)

each example below must show both a preference schedule AND a voting method.

2. David gets more than half the first place votes, but David does not win the election.

11	10
D	M
M	C
C	P
P	D

Note: ONLY Borda count method works here

David has 11 of the 21 votes (more than half).

$\boxed{\text{Borda count Method}}$:

D has $11(4) + 10(1) = 54$
M has $11(3) + 10(4) = 73$ ← Matt wins by Borda count (not David)
C has $11(2) + 10(3) = 52$
P has $11(1) + 10(2) = 31$

3. Peter gets the most last place votes, and Peter wins the election.

11	6	6
P	M	C
C	D	M
D	C	D
M	P	P

Peter has 12 of the 23 last-place votes (more than any other).

Using $\boxed{\text{plurality method}}$ Peter wins: he also has the most 1st place votes.

POSSIBLE EXAMPLES

4. David wins every head-to-head matchup, but David does not win the election. (I.e., more voters prefer David to Matt, and prefer David to Christopher, and prefer David to Peter.)

one example: use the same example as in #2. (check conditions!)

another:

10	9	8
C	M	P
D	D	D
M	C	M
P	P	C

D 9+8 = 17
C 10

D 10+8 = 18
M 9

D 10+9 = 19
P 8

David wins every head-to-head.

using $\boxed{\text{plurality method,}}$ Christopher wins (not David).

(Also possible to find examples using Plur. w/ Elim.)

5. Matt wins. But if Christopher drops out of the race, and people rank the remaining candidates in the same order they did before, Matt doesn't win anymore.

11	10	9
M	D	C
C	M	D
P	C	M
D	P	P

Use $\boxed{\text{plurality method}}$: Matt wins!

But what if Christopher drops out?

11	10	9
M	D	D
P	M	M
D	P	P

Now, using the same method (Plurality), David wins (not Matt).

Notes: (1) for #5 it is possible to find an example with any voting method.

(2) Don't confuse a candidate dropping out of the election, like C did above, with the plurality w/elim voting method. In Plur. w/ Elim, we erase someone if no one got a majority, and look at the ballots without their name. They don't actually leave the election and will still expect to learn results. (Candidates do drop out of elections for their own reasons.)

Once you finish all of these, go back and see if your example works with a different voting method than the one you chose. Try to determine which voting methods are possible, and which are impossible, in each scenario. For example, in problem 1, plurality method is impossible, because plurality method