This worksheet should help you prepare for the first mini-exam. It doesn't have every possible type of question, but it has a good selection. To be sure you're prepared, review all your class notes, worksheets and WeBWork.

Movies showing at a theater in Lexington today include Dunkirk (D), Logan Lucky (L), The Emoji Movie (E), and Spider Man Homecoming (S). A club president has his members rank these movies in order of preference, with the following results:

|       | 9 | 2 | 3 | 3 | 3 |
|-------|---|---|---|---|---|
| 1st   | L | D | S | S | E |
| 2nd   | D | S | D | D | S |
| 3rd   | E | E | E | L | D |
| 4th   | S | L | L | E | L |

1.  If we use the **Plurality method**, which movie will the club see?

    First place votes: L 9, D 2, S 6, E 3

    They will see Logan Lucky (L).

2.  If we use the **Borda count method**, how many Borda points does the movie Spider Man (S) get? Show your work, not just the final answer.

    S: $9(1) + 2(3) + 3(4) + 3(4) + 3(3)$

    $= 9 + 6 + 12 + 12 + 9 = \boxed{48}$

3.  If we used the **Method of Pairwise Comparisons,** find the winner of each of these matchups (if it is a tie, state so). Show a bit of work. (You do not need to finish by computing the pairwise points.)

    L vs D:
    L   9
    Ⓓ   2+3+3+3 = 11

    D vs. E:
    Ⓓ   9 + 2 + 3 + 3 = 17
    E   3

    L vs. E:
    Ⓛ   9 + 3 = 12
    E   2 + 3 + 3 = 8

    D vs. S:
    Ⓓ   9 + 2 = 11
    S   3 + 3 + 3 = 9

    L vs. S:
    L   9
    Ⓢ   2 + 3 + 3 + 3 = 11

    E vs. S:
    Ⓔ   9 + 3 = 12
    S   2 + 3 + 3 = 8

4.  Does this election have a **Condorcet candidate**? If so, which? If not, why?

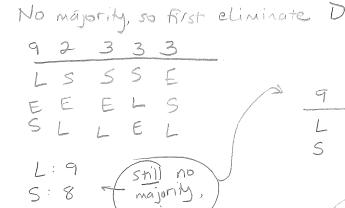    Yes, Dunkirk (D) won all three head-to-head matchups that it was in.

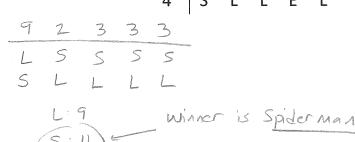5.  Does this election have a **Majority candidate**? If so, which? If not, why?

    Total # voters: 9 + 2 + 3 + 3 + 3 = 20
    # 1st place votes needed for majority: 11
    No candidate has 11 or more first place votes.

6. Actually, the president decided to use the **Plurality with Elimination** method. **Show all the steps clearly** of implementing this method and determine the winner.

|     | 9 | 2 | 3 | 3 | 3 |
|-----|---|---|---|---|---|
| 1st | L | D | S | S | E |
| 2nd | D | S | D | D | S |
| 3rd | E | E | E | L | D |
| 4th | S | L | L | E | L |

No majority, so first eliminate D

| 9 | 2 | 3 | 3 | 3 |
|---|---|---|---|---|
| L | S | S | S | E |
| E | E | E | L | S |
| S | L | L | E | L |

L: 9
S: 8
E: 3

*still no majority, elim E*

| 9 | 2 | 3 | 3 | 3 |
|---|---|---|---|---|
| L | S | S | S | S |
| S | L | L | L | L |

L: 9
S: 11

winner is Spiderman

7. Suppose the president had allowed the club members to rank **all 18 movies** showing at this theater, instead of narrowing it down to four candidates. (Assume the same number of members are voting.)

   a. For the method of pairwise comparisons, **how many different pairs** would we have to check for these eighteen movies? Show both the formula and the answer.
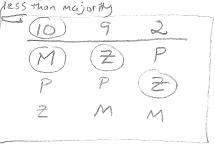
   $$\frac{c(c-1)}{2} = \frac{18(18-1)}{2} = \frac{18(17)}{2} = \boxed{153}$$

   b. For the Borda count method, what is the total number of Borda points in this election? Show the formula.

   #voters $\cdot$ $(1 + 2 + \cdots + c)$     # candidates

   $$\boxed{20 \cdot (1 + 2 + 3 + 4 + 5 + \cdots + 18)}$$
   voters

8. Now suppose we have a different club with **21 members,** and they plan to watch a movie from Netflix instead of going to a theater. They want to choose between The Prestige (P), Zootopia (Z) and Memento (M). **Construct an example** of an election where **Memento would win using plurality,** but **Zootopia would win using plurality with elimination.** Show steps to justify your answer.

One possibility:
(The circled entries are the key to this working)

*less than majority*

| 10 | 9 | 2 |
|----|---|---|
| M  | Z | P |
| P  | P | Z |
| Z  | M | M |

Plurality:

M wins with 10 first place votes.

plurality w/ elim:

majority requires <u>11</u> first place votes, so eliminate P:

| 10 | 9 | 2 |
|----|---|---|
| M  | Z | Z |
| Z  | M | M |

Now Z wins ( Z 11 / M 10 )