

I. ① $y = \frac{5}{x^3} = 5x^{-3}$ $y' = 5(-3)x^{-4} = \boxed{-15x^{-4}}$ either one

② $y = \sqrt[3]{x^{10}} = (x^{10})^{\frac{1}{3}} = x^{10/3}$ $y' = \frac{10}{3} x^{\frac{10}{3}-\frac{2}{3}} = \boxed{\frac{10}{3} x^{\frac{8}{3}}} = \frac{10 \sqrt[3]{x^8}}{3}$

③ $y = \frac{1}{5x^3} = \frac{1}{5} \cdot \frac{1}{x^3} = \frac{1}{5} x^{-3}$ $y' = \frac{1}{5}(-3)x^{-4} = \boxed{-\frac{3}{5} x^{-4}} = \frac{-3}{5x^4}$

④ $y = \frac{7}{6 \sqrt[5]{x^8}} = \frac{7}{6} \cdot \frac{1}{(x^8)^{\frac{1}{5}}} = \frac{7}{6} \cdot \frac{1}{x^{8/5}} = \frac{7}{6} x^{-8/5}$ $y' = \frac{7}{6}(-\frac{8}{5}) x^{-8/5-1} = \boxed{-\frac{56}{30} x^{-13/5}}$

II. ⑤ $y = \frac{x^3 - 3x^2 + 5x + 2}{x^2} = \frac{x^3}{x^2} - \frac{3x^2}{x^2} + \frac{5x}{x^2} + \frac{2}{x^2} = x - 3 + 5x^{-1} + 2x^{-2}$

$y' = 1 - 0 + 5(-1)x^{-2} + 2(-2)x^{-3} = \boxed{1 - 5x^{-2} - 4x^{-3}}$

⑥ $y = x^2(x^3 + \sqrt{x} - \frac{1}{9} + 15) = x^2(x^3 + x^{1/2} - x^{-9} + 15) = x^5 + x^{5/2} - x^{-7} + 15x^2$

$y' = 5x^4 + \frac{5}{2} x^{3/2} + 7x^{-8} + 30x$

don't forget to add exponents.
 $2 + \frac{1}{2} = \frac{4}{2} + \frac{1}{2} = \frac{5}{2}$

III. ⑧ $y = (3x^2 + 2x - 3)(5x^7 + 4x^3 - 2x + 1)$ ← use the product rule "do not simplify"

$y' = (3x^2 + 2x - 3)(35x^6 + 12x^2 - 2) + (5x^7 + 4x^3 - 2x + 1)(6x + 2)$

⑨ $y = \frac{8x^4 + 17}{7x^3 + 2x - 1}$ $y' = \frac{(7x^3 + 2x - 1)(32x^3) - (8x^4 + 17)(21x^2 + 2)}{(7x^3 + 2x - 1)^2}$

IV. ⑩ $y = \frac{5}{\sqrt[3]{3x-5}} = \frac{5}{(3x-5)^{\frac{1}{3}}} = 5(3x-5)^{-\frac{1}{3}}$ (no x's in the numerator, so this is the best form.)

$y' = 5(-\frac{1}{3})(3x-5)^{-\frac{1}{3}-\frac{2}{3}}(3) = \boxed{-\frac{15}{3}(3x-5)^{-1}} = -\frac{5}{3x-5}$

↑ constant stays power rule inside

⑪ $y = (x^3 + 6)^{23}$ $y' = 23(x^3 + 6)^{22} (3x^2) = \boxed{69x^2(x^3 + 6)^{22}}$

power rule inside

⑫ $y = ((x^2 + 1)^4 + 3)^6 + 5x + 10$

$y' = 6((x^2 + 1)^4 + 3)^5 \cdot 4(x^2 + 1)^3 \cdot 2x + 5$

V. (13) (a) $h(x) = \sqrt{f(x)+g(x)} = (f(x)+g(x))^{\frac{1}{2}}$ ← deriv. with respect to x first

$$h'(x) = \underbrace{\frac{1}{2} (f(x)+g(x))^{-\frac{1}{2}}}_{\text{power rule}} \cdot \underbrace{[f'(x)+g'(x)]}_{\text{inside}}$$

↙ next, read values from the given table

$$h'(2) = \frac{1}{2} (f(2)+g(2))^{-\frac{1}{2}} [f'(2)+g'(2)]$$

$$= \frac{1}{2} (5+4)^{-\frac{1}{2}} (7-3) = \frac{1}{2} \cdot 9^{-\frac{1}{2}} (4) = \frac{2}{\sqrt{9}} = \boxed{\frac{2}{3}}$$

(b) $h(x) = f(g(x))$ ← outside function is f ; differentiate it first. then multiply by deriv. of inside function.

$$h'(x) = f'(g(x)) \cdot g'(x)$$

$$h'(2) = f'(g(2)) \cdot g'(2) = f'(4) \cdot (-3) = 9(-3) = \boxed{-27}$$

(14) (a) $h(x) = f(x+g(x)) \Rightarrow h'(x) = \underbrace{f'(x+g(x))}_{\text{outside}} \cdot \underbrace{(1+g'(x))}_{\text{inside}}$

$$h'(2) = f'(2+g(2)) \cdot (1+g'(2))$$

$$= f'(2+(-1)) \cdot (1+(-4)) = f'(1) \cdot (-3) = (-7)(-3) = \boxed{21}$$

(b) $h(x) = \frac{f(x)+4g(x)}{(3x+1)^2}$ use quotient rule! power rule inside

$$h'(x) = \frac{(3x+1)^2 [f'(x)+4g'(x)] - (f(x)+4g(x)) \cdot 2(3x+1) \cdot 3}{(3x+1)^4}$$

$$h'(1) = \frac{(3+1)^2 (f'(1)+4g'(1)) - (f(1)+4g(1)) \cdot 2(3+1) \cdot 3}{(3+1)^4}$$

$$= \frac{4^2(-7+4(\frac{1}{2})) - (6+4 \cdot 1) \cdot 2(4)(3)}{4^4}$$

$$= \frac{4^2(-7+2) - (10)(24)}{256} = \frac{16(-5) - 240}{256} = \frac{-320}{256} = \boxed{-\frac{5}{4}}$$