

I. Find the derivative. **DO NOT SIMPLIFY** your answer.

1. $y = x^4 + x^e + x^\pi + e^x + e^\pi + \ln x + \ln 5$

$$y' = 4x^3 + e x^{e-1} + \pi x^{\pi-1} + e^x + 0 + \frac{1}{x} + 0$$

2. $y = x e^{3x+5}$

$$y' = \underbrace{x}_{\text{copy}} \cdot \underbrace{e^{3x+5}}_{\text{rule inside}} \cdot 3 + \underbrace{e^{3x+5}}_{\text{copy}} \cdot \underbrace{1}_{\text{deriv. of } x} \quad (1)$$

3. $y = \ln(2x^3 + \sqrt{x}) = \ln(2x^3 + x^{\frac{1}{2}})$

$$y' = \frac{1}{2x^3 + x^{\frac{1}{2}}} \cdot (6x^2 + \frac{1}{2} x^{-\frac{1}{2}})$$

those parentheses are important!

4. $y = \frac{e^{4x^2+1}}{13x-8}$

$$y' = \frac{(13x-8) e^{4x^2+1} \cdot 8x - e^{4x^2+1} (13)}{(13x-8)^2} \quad (13)$$

5. $y = 5e^{\sqrt[3]{2x^7-5x+3}} = 5e^{(2x^7-5x+3)^{\frac{1}{3}}}$

$$y' = \underbrace{5}_{\text{e rule}} e^{(2x^7-5x+3)^{\frac{1}{3}}} \cdot \underbrace{\frac{1}{3}}_{\text{power rule}} (2x^7-5x+3)^{-\frac{2}{3}} \cdot \underbrace{(14x^6-5)}_{\text{inside}}$$

II. Find the **second derivative**, $\frac{d^2y}{dx^2}$. You should simplify the first derivative a bit before you differentiate again, but do not simplify the second derivative.

6. $y = (e^{2x} + 8)^7 \quad y' = 7(e^{2x} + 8)^6 \cdot e^{2x} \cdot 2 = 14e^{2x}(e^{2x} + 8)^6$

$$y'' = \underbrace{14e^{2x}}_{\text{copy}} \cdot \underbrace{6}_{\text{power}} (e^{2x} + 8)^5 \cdot \underbrace{e^{2x}}_{\text{e rule}} \cdot \underbrace{2}_{\text{inside}} + \underbrace{(e^{2x} + 8)^6}_{\text{copy}} \cdot \underbrace{14e^{2x}}_{\text{e rule}} \cdot \underbrace{2}_{\text{inside}}$$

7. $y = \ln(3x+1) \quad y' = \frac{1}{3x+1} \cdot 3 = 3(3x+1)^{-1}$

$$y'' = -3(3x+1)^{-2} \cdot 3$$

III. Abstract Functions.

8. Suppose $h(x) = \ln(f(x))$, and the equation of the tangent line to $f(x)$ at $x=2$ is given by $y = 30x - 55$. Find $h'(2)$.

$$h'(x) = \frac{1}{f(x)} \cdot f'(x) \quad \text{so} \quad h'(2) = \frac{f'(2)}{f(2)}$$

$$f'(2) = 30. \quad f(2) = 30(2) - 55 = 5.$$

$$\text{so} \quad h'(2) = \frac{30}{5} = \boxed{6}$$

9. Suppose f and g are differentiable functions which have the following values

- a. Find $h'(1)$ if $h(x) = f(\ln x) + \ln(g(x))$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	16	-9	8	14
1	5	3	7	-6

$$h'(x) = f'(\ln x) \cdot \frac{1}{x} + \frac{1}{g(x)} \cdot g'(x)$$

$$h'(1) = \frac{f'(\ln 1)}{1} + \frac{g'(1)}{g(1)} = \frac{f'(0)}{1} + \frac{-6}{3} = 8 - 2 = \boxed{6}$$

- b. Find $h'(1)$ if $h(x) = x^2 e^{f(x)}$

$$h'(x) = x^2 \cdot e^{f(x)} \cdot f'(x) + e^{f(x)} \cdot 2x$$

$$h'(1) = 1^2 \cdot e^{f(1)} \cdot f'(1) + e^{f(1)} \cdot 2$$

$$= e^5 \cdot 7 + e^5 \cdot 2$$

$$= \boxed{9e^5}$$