1. (a) $\int_0^2 x^3\,dx$

$= \frac{1}{4}x^4 \Big|_0^2$

$= \frac{1}{4}\cdot 2^4 - \frac{1}{4}\cdot 0^4$

$= \boxed{4}$

(b) $\int_0^2 (5x+1)^3\,dx$ $\left[u = 5x+1 \quad \frac{du}{dx} = 5 \quad dx = \frac{1}{5}du \quad \text{If } x=0,\ u=1.\ \text{If } x=2,\ u=11 \right]$

$= \int_1^{11} u^3 \cdot \frac{1}{5}du = \frac{1}{5}\cdot\frac{1}{4}u^4 \Big|_1^{11}$

$= \boxed{\frac{1}{20}\cdot 11^4 - \frac{1}{20}\cdot 1^4} = \frac{1}{20}(11^4 - 1)$

$= \boxed{732}$

2. (a) $\int_0^4 \sqrt{x}\,dx = \int_0^4 x^{\frac{1}{2}}\,dx$

$= \frac{2}{3}x^{3/2}\Big|_0^4 = \frac{2}{3}(\sqrt{x})^3\Big|_0^4$

$= \boxed{\frac{2}{3}\cdot(\sqrt{4})^3 - \frac{2}{3}\cdot(\sqrt{0})^3}$

$= \frac{2}{3}\cdot 8 - 0 = \boxed{\frac{16}{3}}$

(b) $\int_0^5 \sqrt{3x+1}\,dx$ $\left[u = 3x+1 \quad \frac{du}{dx} = 3 \quad dx = \frac{1}{3}du \quad \text{If } x=0,\ u=1.\ \text{If } x=5,\ u=16. \right]$

$= \int_1^{16} \sqrt{u}\cdot\frac{1}{3}du = \int_1^{16} \frac{1}{3}u^{\frac{1}{2}}du$

$= \frac{1}{3}\cdot\frac{2}{3}u^{3/2}\Big|_1^{16} = \frac{2}{9}\cdot(\sqrt{u})^3\Big|_1^{16}$

$= \boxed{\frac{2}{9}\cdot(\sqrt{16})^3 - \frac{2}{9}(\sqrt{1})^3} = \frac{2}{9}(4^3 - 1) = \frac{2\cdot 63}{9} = \boxed{14}$

3. (a) $\int_1^8 \frac{1}{t^5}\,dt = \int_1^8 t^{-5}\,dt$

$= -\frac{1}{4}t^{-4}\Big|_1^8 = -\frac{1}{4t^4}\Big|_1^8$

$= \boxed{\frac{-1}{4\cdot 8^4} - \left(\frac{-1}{4\cdot 1^4}\right)} = \boxed{\frac{4095}{16384}}$

$= \frac{-1}{16384} + \frac{1}{4}$

(b) $\int_1^8 \frac{1}{(6t+3)^5}\,dt$ $\quad u = 6t+3 \quad \frac{du}{dt} = 6 \quad dt = \frac{1}{6}du$ if $t=1,\ u=9.$ If $t=8,\ u=51$

$= \int_9^{51} \frac{1}{u^5}\cdot\frac{1}{6}du = \int_9^{51} \frac{1}{6}u^{-5}du$

$= \frac{1}{6}\cdot\frac{-1}{4}u^{-4}\Big|_9^{51} = \frac{-1}{24u^4}\Big|_9^{51}$

$= \boxed{\frac{-1}{24\cdot 51^4} - \left(\frac{-1}{24\cdot 9^4}\right)}$

4. (a) $\int_1^x \frac{3}{t}\,dt = \int_1^x 3\cdot\frac{1}{t}\,dt$

$= 3\ln|t|\Big|_1^x$

$= \boxed{3\ln|x| - \underbrace{3\ln|1|}_{\to 0}}$

$= \boxed{3\ln|x|}$

(b) $\int_1^x \frac{1}{4t-3}\,dt$ $\left[u = 4t-3 \quad \frac{du}{dt} = 4 \quad dt = \frac{1}{4}du \quad \text{If } t=1,\ u=4-3=1.\ \text{If } t=x,\ u=4x-3 \right]$

$= \int_1^{4x-3} \frac{1}{u}\cdot\frac{1}{4}du = \frac{1}{4}\ln|u|\Big|_1^{4x-3}$

$= \boxed{\frac{1}{4}\ln|4x-3| - \frac{1}{4}\ln 1}$

$= \boxed{\frac{1}{4}\ln|4x-3|}$

5. (a) $\int_{-4}^{2} 5e^{x}dx$

$= 5e^{x}\Big|_{-4}^{2}$

$= \boxed{5e^{2} - 5e^{-4}}$

(b) $\int_{-4}^{2} 5e^{7x+3}dx$ $\left[\begin{array}{l} u = 7x+3 \quad \frac{du}{dx} = 7 \quad dx = \frac{1}{7}du \\ \text{IF } x=-4,\ u=-25.\ \text{IF } x=2,\ u=17. \end{array} \right]$

$= \int_{-25}^{17} 5e^{u} \cdot \frac{1}{7}du = \frac{5}{7}e^{u}\Big|_{-25}^{17}$

$= \boxed{\frac{5}{7}e^{17} - \frac{5}{7}e^{-25}}$

6. (a) $\int_{0}^{2} x^2\left(x^3+7\right)^{10} dx$

$\left[\begin{array}{l} u = x^3+7 \quad \frac{du}{dx} = 3x^2 \quad dx = \frac{1}{3x^2}du \\ \text{IF } x=0,\ u=7.\ \text{IF } x=2,\ u=15 \end{array} \right]$

$\int_{7}^{15} x^2 \cdot u^{10} \cdot \frac{1}{3x^2}du = \int_{7}^{15} \frac{1}{3}u^{10}du$

$= \frac{1}{3}\cdot\frac{1}{11}u^{11}\Big|_{7}^{15} = \boxed{\frac{1}{33}\cdot 15^{11} - \frac{1}{33}\cdot 7^{11}}$

(b) $\int_{-4}^{2} 5xe^{x^2-2}dx$ $\left[\begin{array}{l} u = x^2-2 \quad \frac{du}{dx} = 2x \quad dx = \frac{1}{2x}du \\ \text{IF } x=-4,\ u=(-4)^2-2=14 \\ \text{IF } x=2,\ u=2^2-2=2 \end{array} \right]$

$= \int_{14}^{2} 5xe^{u}\cdot\frac{1}{2x}du = \int_{14}^{2} \frac{5}{2}e^{u}du$

$= \frac{5}{2}e^{u}\Big|_{14}^{2} = \boxed{\frac{5}{2}e^{2} - \frac{5}{2}e^{14}}$

7. $\int_{0}^{x} 2t\sqrt{t^2+1}\,dt$ $\left[\begin{array}{l} u = t^2+1 \quad \frac{du}{dt} = 2t \quad dt = \frac{1}{2t}du \\ \text{IF } t=0,\ u=1.\ \text{IF } t=x,\ u=x^2+1 \end{array} \right]$

$= \int_{1}^{x^2+1} 2t\cdot\sqrt{u}\cdot\frac{1}{2t}du = \int_{1}^{x^2+1} u^{\frac{1}{2}}du = \frac{2}{3}u^{3/2}\Big|_{1}^{x^2+1} = \boxed{\frac{2}{3}(x^2+1)^{3/2} - \frac{2}{3}\cdot 1^{3/2}}$

8. $\int_{0}^{5}\frac{4x}{x^2+1}dx$ $\left[\begin{array}{l} u = x^2+1 \quad \frac{du}{dx} = 2x \quad dx = \frac{1}{2x}du \\ \text{IF } x=0,\ u=1.\ \text{IF } x=5,\ u=5^2+1=26 \end{array} \right]$

$= \int_{1}^{26}\frac{4x}{u}\cdot\frac{1}{2x}du = \int_{1}^{26} 2\cdot\frac{1}{u}du = 2\ln|u|\Big|_{1}^{26}$

$= \boxed{2\ln|26| - \underbrace{2\ln|1|}_{0}} = \boxed{2\ln 26}$