Math 123 Sample Questions for Exam 4 Bonus SOLUTIONS

The following questions should help you prepare for the two bonus questions that will be offered on the final exam. To be sure you will get full credit, you must show clear legible work to support your answer. This is intended just to give a sample; the actual questions may be taken from other material in the course. You should give exact values, not calculator approximations; for example, you'd leave $\sqrt{2}$ in an answer instead of approximating 1.41. Sample solutions are provided in a different file.

1. Find the **average rate of change** of $f(x) = 3x^2 + 5$ from $x = 2$ to $x = 2 + h$.

$$f(2) = 3 \cdot 2^2 + 5 = 3(4) + 5 = 17$$

$$f(2+h) = 3(2+h)^2 + 5 = 3(4 + 4h + h^2) + 5$$

$$= 12 + 12h + 3h^2 + 5$$

$$= 12h + 3h^2 + 17$$

$$\text{AROC} = \frac{f(2+h) - f(2)}{h} = \frac{12h + 3h^2 + 17 - 17}{h}$$

$$= \frac{12h + 3h^2}{h} = \frac{h(12 + 3h)}{h} = \boxed{12 + 3h}$$

2. Find the **derivative** of $f(x) = x^2 e^{\sqrt{5x-3}}$. __DO NOT SIMPLIFY__ your answer.

$$f(x) = x^2 \cdot e^{(5x-3)^{1/2}}$$

$$\boxed{f'(x) = x^2 \cdot e^{(5x-3)^{\frac{1}{2}}} \cdot \frac{1}{2}(5x-3)^{-\frac{1}{2}} \cdot 5 + 2x e^{(5x-3)^{\frac{1}{2}}}}$$

3. Let $f(x) = xe^{-x}$. Find the intervals where $f(x)$ is **concave up**.

$$f'(x) = x \cdot e^{-x}(-1) + e^{-x} = e^{-x}(-x+1)$$

$$f''(x) = e^{-x}(-1) + (-x+1) \cdot e^{-x}(-1)$$

$$= e^{-x}(-1 + x - 1) = e^{-x}(x-2)$$

$f''(x) = 0$ when $x = 2$.

f	$\cap$		$\cup$
f''	$-$		$+$
	test 0	2	test 100

(See example 26 in Chapter 6 lecture notes.)

$\boxed{f \text{ is concave up for } x \text{ in } (2, \infty)}$

4. Let $f(x) = 6x\sqrt{x^2+1}$. Find the **average value** of $f(x)$ over $[0,5]$.

$$\text{Avg. value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\frac{1}{5-0}\int_0^5 6x\sqrt{x^2+1}\,dx$$

$u = x^2+1$

$\frac{du}{dx} = 2x \qquad du = 2x\,dx \qquad dx = \frac{1}{2x}du$

if $x = 0$, $u = 0^2+1 = 1$

if $x = 5$, $u = 5^2+1 = 26$

$$= \frac{1}{5}\int_1^{26} 6x \cdot \sqrt{u} \cdot \frac{1}{2x}\,du$$

$$= \frac{1}{5}\int_1^{26} 3u^{\frac{1}{2}}\,du = \frac{1}{5} \cdot 3 \cdot \frac{2}{3}\,u^{3/2}\Big|_1^{26}$$

$$= \frac{2}{5}u^{3/2}\Big|_1^{26} = \boxed{\frac{2}{5} \cdot 26^{3/2} - \frac{2}{5} \cdot 1^{3/2}}$$

(not required to simplify)