Review of the Four Fairness Criteria

**Majority Criterion**: If candidate X has a majority of the first-place votes, then candidate X should be the winner of the election.
- The majority criterion is always satisfied by the Plurality Method, the Plurality with Elimination Method, and Pairwise Comparison Method.
- The Borda Count Method does not satisfy the majority criterion. This means that the Borda Count Method does not always select the candidate with the majority of first place rankings.

**Condorcet Criterion**: If candidate X is preferred by the voters over each of the other candidates in a head-to-head comparison, then candidate X should be the winner of the election.
- The Condorcet criterion is always satisfied by the Method of Pairwise Comparison.
- The Borda Count Method, the Plurality with Elimination Method, and the Plurality Method might select a Condorcet candidate, but they can also fail to honor the criterion.

**Monotonicity Criterion**: If candidate X is a winner of an election and, in a reelection, the only changes in the ballots are changes that favor X (and only X), then X should still be the winner.
- The Plurality Method, the Borda Count Method and the Pairwise Comparison Method always satisfy the monotonicity criterion.
- The Plurality with Elimination Method can violate the monotonicity criterion.

**Independence of Irrelevant Alternatives Criterion** (IIA): If candidate X is a winner of an election and in a recount one of the nonwinning candidates withdraws or is disqualified, then X should still be the winner.
- The Plurality Method, the Borda Count Method, the Pairwise Comparison Method, and the Plurality with Elimination Method can fail to satisfy the Irrelevant Alternative criterion.

<table>
<thead>
<tr>
<th></th>
<th>Majority</th>
<th>Condorcet</th>
<th>Monotonicity</th>
<th>Ind. of Irr. Alt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plurality</td>
<td>satisfy</td>
<td>violate</td>
<td>satisfy</td>
<td>violate</td>
</tr>
<tr>
<td>Borda</td>
<td>violate</td>
<td>violate</td>
<td>satisfy</td>
<td>violate</td>
</tr>
<tr>
<td>Plur. w/ Elim.</td>
<td>satisfy</td>
<td>violate</td>
<td>violate</td>
<td>violate</td>
</tr>
<tr>
<td>Pairwise comp.</td>
<td>satisfy</td>
<td>satisfy</td>
<td>satisfy</td>
<td>violate</td>
</tr>
</tbody>
</table>

**Arrow’s Impossibility Theorem**: It is mathematically impossible for a democratic voting method to satisfy all four of the fairness criteria.
- Maybe his conditions are too strict? Since Arrow’s original work, many investigators have developed a wide array of desirable fairness rules and showed results similar to Arrow’s theorem.
- In 1972 Arrow was awarded the Nobel Prize in Economics (there is no Nobel Prize in Mathematics) for his pioneering work in what is now known as social-choice theory, a discipline that combines aspects of mathematics, economics, and political science.