1. Define a transformation $T : M_{2\times 2} \to \mathbb{P}_3$ by $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ax^3 + (b+c)x + d$.

   a. Find the kernel of T, a basis for $\ker T$, and $\dim \ker T$.
   Hint: first decide, will the kernel be a collection of polynomials, or of matrices?

   b. Find a basis for the image of T (the range), and its dimension. *Hint*: what polynomials can we possibly get by applying T: *all* of them, or only *some* of them?

2. $\mathcal{B} = \{2t^2 + t, t+1, 3t+2\}$ is a basis for $\mathbb{P}_2$. Let $p = 4t^2 + 5t + 6$. Find $[p]_{\mathcal{B}}$.
   Hint: begin by writing an equation with coefficients c_1, c_2 and c_3 for each basis polynomial, and set them equal to the polynomial p. Then combine like terms.